Kazuhisa Takemura

Behavioral Decision Theory

Psychological and Mathematical Descriptions of Human Choice Behavior

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ISBN 978-4-431-54579-8 ISBN 978-4-431-54580-4 (eBook)
DOI 10.1007/978-4-431-54580-4
Springer Tokyo Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014930167

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

This book provides an overview of the idea of behavioral decision theory and related research findings. Behavioral decision theory is described briefly as the general term for descriptive theories to explain the psychological knowledge related to people's decision-making behavior. It is called theory, but it is a combination of various psychological theories, for which no axiomatic systems such as those with which the utility theory widely used in economics have been established, but it is often limited to qualitative knowledge. As the studies of H. A. Simon, who won the Nobel Prize for economics in 1978, and D. Kahneman, who won the prize in 2002 suggest, however, the psychological methodology and knowledge of behavioral decision theory have been applied widely in such fields as economics, business administration, and engineering, and are expected to become useful in the future.

Behavioral decision theory is related closely to behavioral economics and behavioral finance, which have been popular in recent years. Behavioral economics is an attempt to understand actual human economic behavior, and behavioral finance studies human behavior in financial markets. The research of people's decision-making represents an important part also in these fields, in which various aspects overlap with the scope of behavioral decision theory.

This book covers a range from classical to relatively recent major studies related to behavioral decision theory. It comprises six parts—Part I: Behavioral Decision Theory and the Idea of It (Chap. 1), Part II: Preference Reversal Phenomenon and Description of the Phenomenon (Chaps. 2, 3, and 4), Part III: Expected Utility Theory and Counterexamples (Chaps. 5 and 6), Part IV: Decision Making and Prospect Theory (Chaps. 7 and 8), Part V: The Framing Effect and Its Descriptions (Chaps. 9 and 10), Part VI: Decision-making Process and Its Theory (Chaps. 11 and 12), and Part VII: Behavioral Decision Theory and Good Decision Making (Chap. 13).

Chapter 1 in Part I describes the relations between the decision-making phenomenon and behavioral decision theory. Chapter 2 in Part II explains the phenomenon by which a preference is reversed depending on the mode of making a decision (preference reversal phenomenon), followed by Chap. 3, which addresses the causes of such preference reversal phenomenon, and Chap. 4, which presents the model to explain the psychological process of the preference reversal phenomenon. Subsequently, Chap. 5 in Part III highlights the relation to expected utility theory, which is often used in economics and psychology. Chapter 6 presents consideration of the axioms of expected utility theory and the Allais and Ellsberg Paradoxes. Chapter 7 in Part IV explains the preference paradox and nonlinear utility theory, particularly the prospect theory that Kahneman and others proposed. Chapter 8 introduces prospect theory and cumulative prospect theory, which uses the Choquet integral, and illustrates the decision-making phenomenon that is explainable using this theory. Chapter 9 in Part V introduces the framing effect, in which the decision-making result varies depending on how decision-making problems are described, which is followed by Chap. 10 that presents the theory to explain this framing effect. In Part VI, empirical studies of decision-making processes are introduced in Chap. 11, along with the theories and knowledge of neuroeconomics to explain decision-making processes presented in Chap. 12. Finally in Part VII, theoretical discussions of multiatrribute decision making are introduced using the possibility theorem and other properties of decision theory. Chapter 13 presents a critical examination of the psychological models of multi-attribute decision-making, findings obtained from them, and rational decision-making and considers what constitutes a “good decision.”

Reading this book requires no advanced expertise. Nonetheless, introductory knowledge of psychology, business administration, and economics and approximately high school graduate level mathematics should improve a reader’s comprehension of the content. In addition, each chapter includes a corresponding bibliography, which can be referred to when studying more details related to behavioral decision theory.

The early draft version of this book is based on the series of Japanese articles of *Keizai Seminar* (Economic Seminar). Ms. Fukiko Konishi of Nippon Hyoron Sha, who was responsible for serials of *Keizai Seminar*, provided me with many helpful remarks, related even to small details. Dr. Hajimu Ikeda, Union Press suggested that I should write this book, and provided me with a number of helpful comments for the English manuscript. Mr. Yutaka Hirachi, and Mr. Yoshio Saito, Springer Japan also provided me with valuable comments for the manuscript.

The information provided in this book has been used for lectures at Waseda University, Gakushuin University, Rikkyo University, The University of Tokyo, Tokyo Institute of Technology, Nagoya University, Kansai University, Osaka University of Human Sciences, Kobe University, University of Tsukuba, and Saint Petersburg State University, Russia. Questions and answers exchanged with students at all of those places have contributed greatly to the compilation of this book. Particularly I have received highly valuable opinions from graduate students taking the Takemura Seminar at Waseda University and from researchers in

decision-making studies through usual discussions. Above all, Dr. Yuki Tamari, Mr. Hajime Murakami, Ms. Junko Takeuchi, Ms. Mariko Shinozuka of Waseda University, Mr. Shigetaka Ohkubo of Keio University helped with some of the proofreading and corrections.

Professor Satoshi Fujii at Kyoto University, who has been conducting joint research on decision making for nearly a decade, also provided me with extremely informative advice and suggestions on a regular basis. A part of our joint research is introduced in this book. Professor Hidehiko Takahashi at Kyoto University, Prof. Takayuki Sakagami, Prof. Toshiko Kikkawa, Mr. Shigetaka Ohkubo at Keio University, Prof. Kaori Karasawa at the University of Tokyo, Prof. Henry Montgomery, Prof. Ola Svenson at Stockholm University, Prof. Tommy Gärling at Gothenburg University, Prof. Marcus Selart at the Norwegian School of Economics and Business Administration, Prof. Michael Smithson at the Australian National University, Prof. Yuri Gatanov at Saint Petersburg State University, Prof. Baruch Fischhoff, Carnegie Mellon University, and Prof. Colin Camerer, California Institute of Technology have given me useful comments for our joint research on decision making through daily practice, which also benefitted this book.

In addition, the research discussions and workshops for experimental Social Sciences Project (headed by Prof. Tatsuyoshi Saijo at Osaka University) conducted under a Grant-in-Aid for Scientific Research on Priority Areas of The Ministry of Education, Culture, Sports, Science and Technology (No. 19046007), and for the Prescriptive Social Psychology Project (headed by Kazuhisa Takemura) conducted under a Grant-in-Aid for Scientific Research A (No. 24243061) in which I am currently taking part, have allowed me to exchange opinions with researchers from various fields including experimental psychology, behavioral economics, and experimental economics. I have found those opportunities to be extremely beneficial. I have been participating in the 20-year-old Cognitive and Statistical Decision Making Research SIG (headed by Prof. Kazuo Shigemasu at Teikyo University) from its inception. Moreover, I continue to learn much from researchers in decision-making studies such as Prof. Yutaka Nakamura of University of Tsukuba, Dr. Yasuaki Kobashi of Taikasha and Prof. Kimihiko Yamagishi of Tokyo Institute of Technology, Prof. Kenpei Shiina, Prof. Shuzo Abe, Prof. Mamoru Kaneko, Prof. Tsuyoshi Moriguchi, Prof. Naoto Onzo, Prof. Kazumi Shimizu, and Prof. Shin-ichi Hirota, Dr. Yuki Tamari, Mr. Takashi Ideno, of Waseda University, Prof. Tetsuo Sugimoto of Sophia University, Prof. Mikiya Hayashi of Meisei University, and Prof. Makoto Abe at The University of Tokyo. In addition, Prof. Tsuyoshi Hatori of Ehime University has remained supportive of me through daily discussion related to decision-making research.

I am most appreciative of the guidance and encouragement offered by predecessors such as the late Prof. Sotohiro Kojima (Doshisha University), Prof. Osamu Takagi (Kansai University), Prof. Kazuo Shigemasu (Teikyo University),

Prof. Nozomu Mastubara (Seigakuin University), and Prof. Tomio Kinoshita (International Institute for Advances Studies).

Finally, this book is the fruit of valuable advice from numerous people with whom I have become acquainted but whose names have not been put into print here. I am truly grateful for all of their support.

Shinjuku-ku, Tokyo, Japan Kazuhisa Takemura

About the Author

Kazuhisa Takemura is a Japanese psychologist and an educator who holds many academic positions. Besides being active as a professor, Department of Psychology, Waseda University, he is also engaged as a director of the Institute for Decision Research, Waseda University, as a professor of Waseda MBA school, as well as a research fellow of the Waseda Research Institute for Science and Engineering.

He received his BA and MA from the Department of Psychology, Doshisha University in 1983 and 1985, and received his Ph.D. (System Science) from Tokyo Institute of Technology in 1994, and Ph.D. (Medical Science) from Kitasato University in 2013.

He also worked abroad as a visiting researcher (James Cook University (Australia), La Trobe University (Australia), Australian National University (Australia), Tinbergen Institute (the Netherlands), Gothenburg University (Sweden), and Stockholm University (Sweden). He was also a Fulbright Senior Researcher at the Department of Social and Decision Science, Carnegie Mellon University (USA) from 1999 to 2000, and Visiting Professor at the Department of Psychology, St. Petersburg State University (Russia) in 2008.

His main research topic is human judgment and decision making, especially mathematical modeling of preferential judgment and choice. He received Hayashi Award (Distinguished Scholar) from The Behaviormetric Society of Japan in 2002, Excellent Paper Award from Japan Society of Kansei Engineering in 2003, and the Book Award from Japanese Society of Social Psychology in 2010.

He has track record in teaching behavioral decision theory, marketing, economic psychology, consumer behavior, social psychology, and psychometrics in many universities (Tokyo University, University of Tsukuba, Kobe University, Nagoya University, Tokyo Institute of Technology, Gakushuin University, Rikkyo University, Tokyo International University, St. Petersburg State University).

Contents

Part I
Behavioral Decision Theory and the Idea of It

Chapter 1
Decision-Making Phenomenon and Behavioral Decision Theory

This chapter first presents the conceptual framework of the decision-making phenomenon, which is apparently almost universal among decision-making studies and which explains how decision-making under certainty, risk, and uncertainty can be understood. Consequently, the way in which behavioral decision theory is positioned in the study of decision-making and how it is related to other theories of decision-making are explained.

1 What Is Decision-Making?

Although decision-making broadly refers to the function of consciousness to make a decision, it can also be defined technically as the act of selecting an alternative from a group of alternatives, i.e., the choice of action (Takemura 1996). Selecting a preferred means of transportation, deciding which product to purchase, and determining which proposal to adopt are examples of decision-making. We make decisions as consumers about purchasing various goods. At times, we must make decisions related to corporate activities and political issues.

At this point, we review the decision-making phenomenon using the concept of aggregation. Let A denote a finite set of alternatives. Its elements are organized into mutually exclusive alternatives $a_1, \ldots, a_i, \ldots, a_l$ (l is the number of alternatives), which can be described as the set $A = \{a_1, \ldots, a_i, \ldots, a_l\}$ (Although set A can be assumed as an infinite set, it is considered a finite set in this case for simplification. The following is also treated as a finite set to simplify the expressions). The method of defining a set by enumerating its components in this way is called extensional definition. For instance, the elements of A can be interpreted as the alternatives consisting of stocks invested, which are $a_1, \ldots, a_i, \ldots, a_l$.

Subsequently, we consider the set of outcomes from choosing these alternatives as $X = \{x_1, \ldots, x_j, \ldots, x_m\}$. For instance, the elements of X include x_1 = a loss of \$10,000, x_2 = neither a gain nor a loss, and x_3 = a gain of \$10,000. When a specific alternative a_i is adopted, an outcome, x_j is expected to appear. However, a_i and x_j

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_1,

do not necessarily mutually correspond on a one-to-one basis. In fact, a_i and x_j mutually correspond in many cases with some uncertainty. In other words, the result x_j from adopting the alternative a_i can be thought to depend at least on conditions such as the state of nature, as $\Theta = \{\theta_1, \ldots, \theta_k, \ldots, \theta_n\}$. The examples include θ_1 = a decline in interest rates, θ_2 = maintenance of the current conditions, and θ_3 = an increase in interest rates. Although, in general, a decrease in interest rates causes an increase in stock prices, stock price fluctuations vary depending on the stock. They are also affected by factors other than the level of interest rates. In this case, however, we simplify the situation and consider that there are various stocks. We assume that the stock trading conditions are determined by changes in interest rates, as shown in Table 1.1. Consequently, as presented in Table 1.1, the result of purchasing a stock is determined by interest rates.

Table 1.1 Example of a decision problem

	θ		
A	θ_1: Decline in interest rates	θ_2: Maintenance of status quo	θ_3: Rise in interest rates
a_1: Brand 1	x_3: \$10,000	x_2: \$0	x_1: −\$10,000
a_2: Brand 2	x_1: −\$10,000	x_2: \$0	x_3: \$10,000
a_3: Brand 3	x_3: \$10,000	x_3: \$10,000	x_1: −\$10,000

The information presented in Table 1.1 suggests that the outcome is determined by the function (mapping) from the alternative selected and conditions related to the outcome, which is

$$f : A \times \Theta \to X.$$

In this case, A × Θ is a Cartesian product representing the set of the possible combinations of the set A and the set Θ. Although the sign is the same as that for arithmetic multiplication, the meaning differs. The set is expressed as

$$A \times \Theta = \{(a_i, \theta_k) | a_i \in A, \theta_k \in \Theta\}.$$

This equation states that the set with elements including all pairs (a_i, θ_k) that incorporates the order of an arbitrary element a_i of set A and an arbitrary element θ_k of set Θ is specified as $A \times \Theta$. Such a method of defining the set based on the properties of the elements of the set is called an intentional definition. Furthermore,

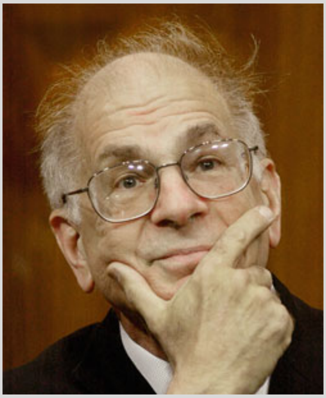
$$f : A \times \Theta \to X$$

is an expression showing that when the alternative a_i and the state θ_k are specified, then one of the results, x_j is determined. As described later, in actual decision-making, which elements in the set of Θ appear and what the elements of the set of Θ actually are remain uncertain in many cases.

2 Structure of Preference Relations and Decision-Making Problems

Daniel Kahneman

Born in 1934. Graduated from the Hebrew University of Jerusalem, he earned a Ph.D. from The University of California. Currently working as a Professor at Princeton University. Awarded the Nobel Memorial Prize in Economic Sciences in 2002 for his achievements in the application of psychological studies—including decision-making and judgment under uncertainty—to economics.



Photograph: AP/Aflo

The element of the set of outcomes is money in the case of Table 1.1. It might be apparent, in general, that the state of no gain and no loss (x_2) is preferred to a loss of \$10,000 (x_1), and that a gain of \$10,000 (x_3) is preferred to the state of no gain and no loss (x_2). However, we will intentionally express these relations. Letting $A \succ B$ mean "A is preferred to B", then the preference relations can be expressed as $x_3 \succ x_2$, $x_2 \succ x_1$, and $x_3 \succ x_1$. The result is simply a state. Cases other than those that can be expressed with money exist; there might also be peculiar individuals who would not mind the difference between \$10,000 and \$11,000. Therefore, inquiring into the preference relations that are associated with the results is an important undertaking.

The simplest preference relation is a binary relation for which one of two alternatives is preferred. We consider the preference relations of a decision-maker in connection with the elements of the set of the results, X. In other words, we assume such relations as "which one is preferred" and "both are equally attractive," that are empirically observed. We express them using the symbol, $\succsim$. When we assuming a relation of "which one is preferred", we express it using symbol $\succ$. When the set that has collected all ordered pairs (x_i, x_j) that become $x_i \succsim x_j$ is R, it can be expressed as

$$R = \left\{ \left(x_i, x_j\right) \middle| x_i \succsim x_j, x_i, x_j \in X \right\}.$$

This set R forms a subset of the Cartesian product set of the set of results X, which expresses the binary relation of X. In other words,

$$R \subset X \times X = \left\{ \left(x_i, x_j\right) \middle| x_i, x_j \in X \right\}$$

holds true.

A set that specifies a set and its relations and other properties is called a relational system. The set $\langle X, R \rangle$ consisting of binary relations on the set, X, is also a relational system. It is called a preference structure because it is a relational system related to preference (Ichikawa 1983).

In summary, the five-tuple including the set of alternatives A, the set of states Θ, the set of results X, mapping $f: A \times \Theta \rightarrow X$, and preference structure $\langle X, \succsim \rangle$ can be regarded as minimally expressing the problem of decision-making that depends on the state. In the theory of decision-making, this set $\langle A, \Theta, X, f, \langle X, \succsim \rangle \rangle$ is often expressed as "decision-making problems" (or, sometimes "decision problems under uncertainty"). In addition, in cases of interactive conditions and social decision-making that assume multiple decision-makers, if the set of decision-makers is I and the alternatives, A^i, and preference relations, R^i, vary depending on $i \in I$, then the decision-making problem can be expressed as $\langle A^1, A^2, \ldots, A^n, \Theta, X, f, \langle X, \succsim^1 \rangle, \langle X, \succsim^2 \rangle, \ldots, \langle X, \succsim^n \rangle \rangle$ (Ichikawa 1983). This example assumes that the alternatives and preference relations of the decision-makers differ, and that the mapping f to the state Θ and the set of results X is common among all members. However, the description of decision-making problems that vary among individuals is also possible.

3 Decision-Making and Uncertainty

Although it might be a simple term, decision-making takes various forms. Decision-making cases are broadly categorized into three groups, as presented in Fig. 1.1, based on the characteristics of the knowledge of the decision-making environment, i.e., how much the decision-makers know about their surrounding environment. The following describes each of the three.

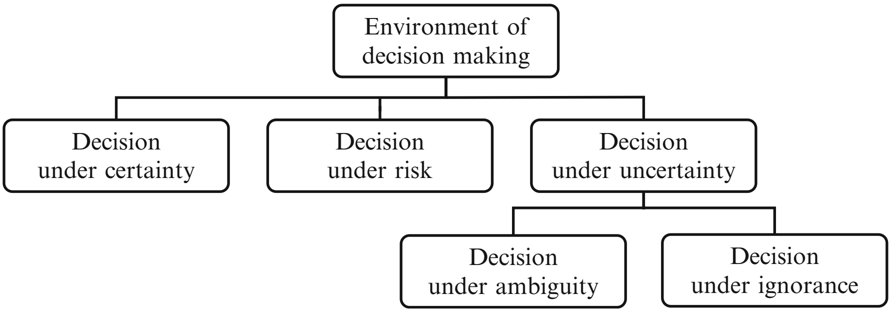


Fig. 1.1 Taxonomy of uncertainties as decision environment

3.1 Decision-Making Under Certainty

The first is decision-making under certainty, which is the type of decision-making in a situation where the result of selecting an alternative is certainly determined. For instance, a situation to decide whether to use $50 in cash or $60 in gift certificates for a purchase is decision-making under certainty. If explained using the expressions with the sets described earlier, then it is the case in which θ_k of the set of states, Θ, will occur is already known or the result, x_j, is determined solely by the alternative, a_i, irrespective of the condition of θ_k (even with an arbitrary θ_k). Another expression is that it is the case in which mapping, $g : A \to X$, from the set of alternatives A to the set of results X exists.

3.2 Decision-Making Under Risk

The second is decision-making under risk. Although risk has broader meanings such as "danger" and "a loss" in the field of psychology and risk analysis (Hirota et al. 2002), in the context of decision-making research, it refers to a condition that occurs with known probability as the result of selecting an alternative. We consider, for example, decision-making as whether to take an umbrella or not. When the prospects for rain can be expressed as a probability, the decision of whether to take an umbrella or not constitutes decision-making under risk. Additionally in this case, although the value of taking an umbrella is high if it rains, it will be merely inconvenient if it does not rain. In this way, the result of choosing an alternative can be regarded as depending on conditions such as the weather.

For organizing decision-making under risk using the expression of sets, we assume a situation in which a probability distribution is defined on the set of states Θ. In the case presented earlier, for instance, we assume that the probability distribution is known to be the probability $p(\theta_1) = 0.3$ of a decrease in interest rates, θ_1, the probability $p(\theta_2) = 0.5$ of maintaining the current situation, θ_2, and the probability $p(\theta_3) = 0.2$ of an increase in interest rates, θ_3. Consequently, the probability of the result X for each alternative a_i can be determined as presented in Table 1.2. The results in Table 1.2 were converted based on results presented in Table 1.1 based on the probability distribution on Θ. We specifically address, for instance, the p_{33} part in Table 1.2, which is the probability of the result (x_3), which is a gain of $10,000 when Stock 3 is selected. Table 1.1 indicates that this result occurs when the states θ_1 and θ_2 take place. Consequently, the probability, p_{33}, is $p(\theta_1) + p(\theta_2) = 0.3 + 0.5 = 0.8$. Therefore $p_{33} = 0.8$, as presented in Table 1.2.

Table 1.2 Example of probability distributions of the outcomes decision making under risk

	X		
A	x_1: −$10,000	x_2: $0	x_3: $10,000
a_1: Brand 1	p_{11}: 0.2	p_{12}: 0.5	p_{13}: 0.3
a_2: Brand 2	p_{21}: 0.3	p_{22}: 0.5	p_{23}: 0.2
a_3: Brand 3	p_{31}: 0.2	p_{23}: 0.0	p_{33}: 0.8

Consequently, the problem of decision-making under risk, i.e., which element of the alternative set A is selected, can be replaced with the problem of which of the probability distribution $p_1 = [p_{11}, p_{12}, \ldots, p_{1m}]$, $p_2 = [p_{21}, p_{22}, \ldots, p_{2m}]$, and $p_l = [p_{l1}, p_{l2}, \ldots, p_{lm}]$ on X is selected. Therefore, decision-making under risk can be expressed using the preference structure, $\langle P, \succsim \rangle$, in which the preference relation, $\succsim$, is added to the set $P = \{p_1, p_2, \ldots, p_l\}$ of probabilities of X.

3.3 Decision-Making Under Uncertainty

Finally, the third group is decision-making under uncertainty. Uncertainty in this context refers to a state in which the probability of the result of selecting an alternative is not known. Decision-making under such uncertainty can be sub-classified as follows (Takemura 1996). The first group is decision-making under ambiguity. This ambiguity refers to a state in which, although the condition and results that will occur are known, the probabilities of the condition and results to occur are unknown.

When expressed as sets, it is a state in which all the elements θ_i of the state Θ and the elements x_j of set X of the results are already known. However, the probability distribution on Θ or that on X is unknown. In the earlier example, we assume that the probability $p(\theta_1)$ of a decrease in interest rates, θ_1, the probability $p(\theta_2)$ of maintaining the current condition, θ_2, and the probability $p(\theta_3)$ of an increase in interest rates, θ_3, are all either ambiguously known or unknown. Consequently, the probability of the set of the results of adopting the selected alternatives becomes either ambiguous or unknown, as presented in Table 1.3.

Table 1.3 Example of unknown probability distribution of the putcomes in decision making under ambiguity

	X		
A	x_1: −$10,000	x_2: $0	x_3: $10,000
a_1: Brand 1	p_{11}: Unknown	p_{12}: Some	p_{13}: Unknown
a_2: Brand 2	p_{21}: Unknown	p_{22}: Some	p_{23}: Unknown
a_3: Brand 3	p_{31}: Fairly low	p_{23}: Fairly low	p_{33}: Probably high

In this case, the probability cannot be expressed in numerical values; instead, linguistic expressions such as "probably high," "rather low," and "moderate" might be used. In fact, even weather forecast experts who have received natural science training are found to have the tendency of using language rather than numerical values to express probabilities (Beyth-Marom 1982). Furthermore, even if uncertainty is expressible in numbers, measures such as probability that do not satisfy additivity (e.g., measure of possibility and Dempster–Shafer measure) might be used (Smithson 1989; Takemura 2000).

The second category of decision-making under uncertainty is decision-making under ignorance when the elements of the set of states or the elements of the set of results are unknown (Smithson 1989; Smithson et al. 2000), which refers to, for instance, a state in which the situation and results, and even the possibility of the results as a consequence of adopting a social policy are unknown. Therefore, when expressed in sets, the state in which the element θ of the set Θ of states and the element x of the set X of results are not known.

Decision-making under ignorance includes cases in which the range of alternatives, possible states, and the range of results are not clearly known. Such cases of decision-making under ignorance occur frequently as exemplified by the various types of decisions made in an unknown land in the actual society. When the level of ignorance is high, not only the elements of the set X of results, the set A of alternatives, and the set Θ of states, but the ignorance of the entire set itself might also occur. This ignorance can be categorized further depending on which of X, A, and Θ is unknown. Currently, however, theories that can accommodate such a case of decision-making under ignorance in which the entire set is unknown are nearly nonexistent.

4 Approaches to Decision-Making Research and Behavioral Decision Theory

Herbert A. Simon

Born in 1916. Graduated and earned a Ph.D. in political science from the University of Chicago. After serving at Illinois Institute of Technology and Carnegie Institute of Technology, he worked as a professor at the School of Computer Science and Psychology Department at Carnegie Mellon University and died in Pittsburgh in 2001. He made important contributions to widely various fields including psychology, computer science, business administration, political science, and economics. In 1978, he was awarded the Nobel Memorial Prize in Economic Sciences for his achievements in the research of human decision-making process in organizations.

Photograph: AP/Aflo

As might be evident from the discussion presented above, the same decision-making takes various forms. What are the theoretical frameworks that could be used to explain the decision-making phenomenon? Although numerous theories related to decision-making have been developed, they are, in essence, often broadly divided into two types: normative theory and descriptive theory (Hirota et al. 2002; Kobashi 1988; Sayeki 1986). The former is a theory that is intended to support rational decision-making. It is a platform used to assess a desirable form of decision-making. The latter is a theory that describes how people actually make decisions.

Both normative and descriptive theories reflect the nature of actual human decision-making to a degree. Even descriptive theory seeks a certain level of rationality in actual human decision-making. Consequently, the two are mutually indistinguishable. Nonetheless, a major example of normative theory is regarded as the system of utility theory that is widely used in economics. A salient example of descriptive theory is probably behavioral decision theory. Utility theory has numerous variations. Most theories have established axioms and mathematically developed principles (Edwards 1992; Fishburn 1988; Barberá et al. 1998). In contrast, behavioral decision theory covers a considerably wide range of variations of theoretical expressions, including theories that have been developed mathematically and those expressed only with natural language (Edwards 1961; Hirota et al. 2002; Poulton 1994; Sayeki 1986; Takemura 1996; Wright 1985).

In the study of decision-making, the study of normative theory has traditionally preceded the other. A comparison between theoretical and actual human decision-making activities has engendered the research of behavioral decision theory, which is a descriptive study of theories (Kobashi 1988). When asked who the founder of behavioral decision theory is, many people might think of Simon or Kahneman: psychologists who were awarded the Nobel Prize for Economic Sciences. However, W. Edwards actually originated the theory (Sayeki 1986). He began his psychological research in 1948 (http://www.usc.edu/dept/LAS/psychology/people/edwards.html) and had written a review article entitled *Behavioral Decision Theory* already in 1961 (Edwards 1961).

The study of behavioral decision theory has traditionally been conducted in the field of psychology. The methodology particularly can be regarded as having been developed by mathematical psychologists and experimental psychologists who are divided according to research fields into cognitive psychologists and social psychologists. Some fields of study are affected by behavioral decision theory. They have become more popular in recent years, including fields such as behavioral economics (e.g., Tada 2003; Camerer et al. 2004) and behavioral finance (e.g., Goldberg and von Nitzsch 2001). Additionally, recent years have shown a trend of adopting certain characteristics of descriptive theory into normative theory as opposed to a one-sided relation in which normative theory studies precede descriptive theory studies (Camerer et al. 2004; Edwards 1992; Fishburn 1988).

Finally, an approach called a prescriptive approach exists as the third one after normative theory and descriptive theory (Bell et al. 1988). The term "prescriptive" is derived from prescriptions issued by physicians. The prescriptive approach aims to support rational decision-making according to the actual conditions of problems. In actual decision-making problems such as social consensus-building and management decision-making, strict normative theory cannot be established in some cases because of uncertainty including ambiguity and ignorance, rendering the approach using normative theory unfeasible. Sole dependence on description, as the descriptive theory espouses, might not engender problem-solving. Accordingly, this approach is extremely important considering the support for decision-making in solving real problems. The knowledge of behavioral decision theory that describes how people make decisions in reality is expected to facilitate the adoption of this approach considerably.

Behavioral Decision Theory and Psychologists

The answer to the question of whether behavioral decision theory is a major part of the field of psychology is that it is slightly less dominant than clinical psychology and other fields of experimental psychology. If asked whether the founders of the theory, Edwards, Simon, and Kahneman, are very well-known psychologists among others, the answer is that, actually, they are not: clinical psychologists tend to be interested in such matters as abnormal psychology and personality; experimental psychologists tend to examine detailed experimental techniques more specifically. Moreover, the fact that, unlike economics, many psychologists are traditionally interested in the results of experiments and research rather than theory might be somehow relevant.

In recent years, however, research of behavioral decision theory has become increasingly popular in the U.S. and Europe. Academic conferences related to behavioral decision theory include the European Association for Decision Making (EADM: http://eadm.eu/) in Europe and the Society for Judgment and Decision Making (SJDM: http://www.sjdm.org/) in the U.S. Both consist principally of psychologists. Although no academic conferences for behavioral decision theory exist in Japan, a study group called the Cognitive and Statistical Decision Making Research SIG, whose regular meetings (research paper presentations) are held a few times a year at the University of Tokyo, Tokyo Institute of Technology, and Waseda University on a rotating basis (representative, Kazuo Shigemasu; managers, Kimihiko Yamagishi and the author). Announcements of regular meetings and other information are posted on the website of Dr. Yasuaki Kobashi (http://homepage3.nifty.com/hiway/dm/d-conts.htm). Any interested person might participate in the meetings (no particular restrictions on participation apply). In addition, this website introduces documents related to behavioral decision theory, which might be used as a reference by readers who are interested in behavioral decision-making theory.

References

Barberá, S., Hammond, P. J. and Seidl, C. (Eds.). (1998). *Handbook of utility theory*. Vol.1 (Principles), Dordrecht, The Netherlands: Kluwer Academic Publishers.

Bell, D. E., Raiffa, H., & Tversky, A. (1988). Descriptive, normative, and prescriptive interactions in decision making. In D. E. Bell, H. Raiffa, & A. Tversky (Eds.), *Decision making: Descriptive, normative, and prescriptive interaction* (pp. 9–30). New York: Cambridge University Press.

Beyth-Marom, R. (1982). How probable is probable?: A numerical translation of verbal probability expressions. *Journal of Forecasting, 1*, 267–269.

Camerer, C. F., Loewenstein, G., & Rabin, M. (Eds.). (2004). *Advances in behavioral economics*. Princeton: Princeton University Press.

Edwards, W. (1961). Behavioral decision theory. *Annual Review of Psychology, 12*, 473–498.

Edwards, W. (Ed.). (1992). *Utility theories: Measurement and applications*. Boston: Kluwer Academic Publishers.

Fishburn, P. C. (1988). *Nonlinear preference and utility theory*. Sussex: Wheatsheaf Books.

Goldberg, J., & von Nitzsch, R. (2001). *Behavioral Finance*, New York: Wiley. (Makabe, A. (trans.) (2002). *Behavioral Finance: A New Financial Theory to Explain the Irrationality of the Market*, Tokyo: Diamond Inc.)

Hirota, S., Masuda, S., & Sakagami, T. (Eds.). (2002). *Shinrigaku ga Egaku Risuku no Sekai - Kodoteki Ishikettei Nyumon [World of risk drawn by psychology: An introduction to behavioral decision-making]*. Tokyo: Keio University Press.

Ichikawa, A. (1983). *Ishiketteiron: Enjiniaringu Saiensu Koza33 [Decision-making theory: Engineering science course 33]*. Tokyo: Kyoritsu Publishing.

Kobashi, Y. (1988). *Ninchi Kagaku Sensho 18: Kettei wo Shien Suru [Selected works in cognitive science 18: Support decisions]*. Tokyo: University of Tokyo Press.

Poulton, E. C. (1994). *Behavioral decision theory: A new approach*. New York: Cambridge University Press.

Sayeki, Y. (1986). *Ninchi Kagaku no Hoho: Ninchi Kagaku Sensho 10 [Methods of cognitive science: Cognitive science 10]*. Tokyo: University of Tokyo Press.

Smithson, M. (1989). *Ignorance and uncertainty: Emerging paradigms*. New York: Springer.

Smithson, M., Bartos, T., & Takemura, K. (2000). Human judgment under sample space ignorance. *Risk, Decision and Policy, 5*, 135–150.

Tada, Y. (2003). *Kodo Keizaigaku Nyumon [Introduction to behavioral economics]*. Tokyo: Nikkei Inc.

Takemura, K. (1996). Ishikettei to Sono Shien [Decision-making and support for decision-making]. In S. Ichikawa (Ed.), *Ninchi Shinrigaku 4kan Shiko [Cognitive psychology vol. 4 thoughts]* (pp. 81–105). Tokyo: University of Tokyo Press.

Takemura, K. (2000). Vagueness in human judgment and decision making. In Z. Q. Liu & S. Miyamoto (Eds.), *Soft computing for human centered machines* (pp. 249–281). Tokyo: Springer.

Wright, G. (Ed.). (1985). *Behavioral decision making*. New York: Plenum.

Part II
Preference Reversal Phenomenon and Description of the Phenomenon

Chapter 2
Ordinal Utility and Preference Reversal Phenomenon

Chapter 1 described that decision-making phenomena are broadly divisible into those under certainty, those under risk, and others under uncertainty. This chapter first introduces the concept of utility used to explain such decision-making phenomena and briefly presents the idea of traditional ordinal utility theory.

Ordinal utility theory is assumed in many theories in economics. However, it includes some phenomena that constitute counterexamples that cannot be justified from the perspective of behavioral decision theory. Among such cases, this chapter presents specific examination of a case in which the transitivity premised on ordinal utility is not satisfied, and presents a phenomenon known as preference reversal. Transitivity refers to a relation between two alternatives, which is a consistent attribute of preference relation that, for instance, if oranges are preferred to bananas and apples are preferred to oranges, then apples are preferred to bananas. In addition, preference reversal is regarded as a phenomenon that deviates from the procedural invariance that preference cannot be reversed by the preference revelation procedures. In this type of case, for instance, when making a purchase decision, although Brand A is said to be more desirable than Brand B when their values (e.g., the prices considered reasonable) are assessed independently, Brand B is chosen if the two are actually compared.

1 What Is Utility?

Utility is interpreted in daily usage as the subjective value or desirability of a result of selecting an alternative. In decision-making theory, it is often technically considered a real-valued function to express a preference relation; it is therefore sometimes called a utility function. Utility is considered in terms of real numbers because the mathematical analysis of a decision-making phenomenon provides the benefit of facilitating the prediction and explanation of the phenomenon. Expressions using utility are also used in decision-aid based on technologies using computers and other tools that are designed to support decisions with which

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_2,

decision-makers can be satisfied. Such technologies have yielded practical advances (Kobashi 1988).

The following presents some simple examples of utility. We consider a case of decision-making under certainty, in which a product—either Brand A or Brand B—is to be selected. In this case, utility refers to the relative preference for Brand A to Brand B (Brand A $\succsim$ Brand B). Only in such a case is the real number that the utility of Brand A ($u(Brand\ A)$) higher than the utility of Brand B ($u(Brand\ B)$). In other words, when a relation

$$u(Brand\ A) \ \geq \ u(Brand\ B) \Leftrightarrow Brand\ A \succsim Brand\ B.$$

holds true, then the preference relation is expressed with a utility function u. Particularly, the type of utility that maintains only the order of preference is called ordinal utility. Ordinal utility does not lose its fundamental meaning even if monotonic increasing conversion of its utility function is performed; it corresponds to the ordinal scale used in psychology and statistics. For instance, if u represents ordinal utility, then the preference relation is maintained even if, for example, $u(Brand\ A) = 5$ and $u(Brand\ B) = 2$ are changed to $\phi(u(Brand\ A)) = 8$ and $\phi(u(Brand\ B)) = 3$ using the function ϕ that increases the values monotonically.

The following expresses ordinal utility slightly more formally. Assuming that Set A of alternatives is finite nonempty and that the preference structure $\langle A, \succsim \rangle$ is a weak order, then the preference structure refers to the set that combines the set of alternatives and a preference relation $\succsim$ of some kind. The weak order in this case represents the relation in which the following two conditions hold:

1. Comparability $\forall\ x, y \in A, x \succsim y \vee y \succsim x.$
   In other words, this is such a relation in which x, y ($\forall\ x, y \in A$), $x \succsim y$, or $y \succsim x$ of Set A of alternatives exists. In this case, the symbol $\vee$ is a logical symbol for "or," which means that at least one of them holds true. Comparability is also called connectedness or completeness. For instance, if the set of brands considered is A and $x \succsim y$ is interpreted as a relation by which y (Brand y) is preferred to x (Brand x) or is interpreted as indifference, then this is a case that can be determined as one in which Brand x is preferred to Brand y or indifference or one in which Brand y is preferred to Brand x or indifference. A situation for which it is unknown "which one is preferred, or whether the chooser is indifferent" does not satisfy comparability.
2. Transitivity $\forall\ x, y, z \in A, x \succsim y \wedge y \succsim z \Rightarrow x \succsim z.$
   In other words, this is a relation in which, if $x \succsim y$ and $y \succsim z$, $x \succsim z$ holds for the arbitrary elements x, y, z ($\forall\ x, y, z \in A$) of A. In this case, the symbol $\wedge$ represents a logical symbol for "and," which means that both relations hold true. For instance, if A is a set of alternatives of product brands just as in the example presented above and $x \succsim y$ is interpreted, then the transitivity is satisfied if there is a relation by which Brand x is preferred to Brand z or indifference when Brand x is preferred to Brand y or indifference or Brand y is preferred to Brand z or indifference.

If transitivity does not hold, it is a three-cornered deadlock relation. For instance, if the power relation of rock–paper–scissors is $\succ$, then rock $\succ$ scissors and scissors $\succ$ paper, but not rock $\succ$ paper. Consequently, $\succ$ does not satisfy transitivity.

We know that the following theorem holds for the weak order that satisfies these two characteristics (Krantz et al. 1971).

The theorem related to a weak order on a finite set (Krantz et al. 1971)

If a preference structure of a finite nonempty set A, $\langle A, \succsim \rangle$, is a weak order, then there exists a real-valued function (ordinal utility function) $u : A \to Re$ on A such that for all $x, y \in A$,

$$x \succsim y \Leftrightarrow u(x) \geq u(y).$$

In other words, this theorem means that if the preference such as a weak order is being made, then it can be expressed with a function that takes real numbers that maintain the preference relation. Therefore, it indicates that the preference relation of a qualitative weak order can be examined by quantifying it using ordinal utility. Although this theorem is based on a finite set in this case, we know that it also applies to a countably infinite set and further to an uncountably infinite set with certain conditions added (Krantz et al. 1971).

In addition, the following theorem holds for ordinal utility (Krantz et al. 1971).

The theorem related to weak order uniqueness on a finite set (Krantz et al. 1971)

If the preference structure $\langle A, \succsim \rangle$ on a finite nonempty set A is a weak order, then $\langle A, \succsim \rangle$ is expressed as $\langle Re, \geq \rangle$ through the real-valued function $u : A \to Re$ of A indicated in the theorem above; the structure $\langle \langle A, \succsim \rangle, \langle Re, \geq \rangle, u \rangle$ becomes an ordinal scale.

Although this theorem assumes a finite set, we know that it applies also to countably and uncountably infinite sets (Krantz et al. 1971).

Aside from ordinal utility, cardinal utility, which is often used in economics, is a type of utility that does not lose its fundamental meaning even with an interval scale, i.e., positive linear transformation (a linear transformation of multiplication by a constant and adding with a constant), as used in psychology and statistics. Using cardinal utility,

$$\forall x, y \in A, x \succsim y \Leftrightarrow u(x) \geq u(y)$$
$$\Leftrightarrow \phi(u(x)) \geq \phi(u(y))$$

where $\phi(u(x)) = \alpha u(x) + \beta (\alpha > 0)$ holds true.

2 Does a Weak Order Empirically Hold?

Does the weak order assumed by ordinal utility theory and by cardinal utility theory hold in actual decision-making?

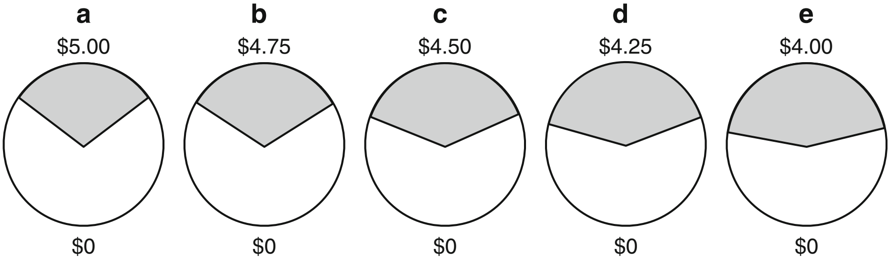


Fig. 2.1 Examples of gambling cards used in the experiment. *Source*: Tversky (1969). Reproduced in part by author

Tversky (1969) used experiments to examine whether the transitivity assumed by weak orders was satisfied in decision-making. He presented two cards with a pie chart test to subjects as presented in Fig. 2.1. He then asked the subjects which *gamble* they would prefer. The respondents were not allowed to express an indifferent preference relation. They were requested to indicate which was preferred. Therefore, this choice indicates a strong preference relation $x \succ y$, i.e., $x \succsim y \wedge$ not $(y \succsim x)$ (where $\succsim$ is a weak order). The cards had the amount of prize money written above the pie charts. The percentage of the area of the black sector in the circle was presented as the winning percentage.

Although multiple patterns were prepared in the experiment, the typical pattern was that two of five cards as in Table 2.1 were combined and the respondents were asked which one they would prefer. The winning percentage rose and the amount of prize money decreased while moving from a to e. In the case of comparative judgments such as those between a and b and between b and c, a slight difference in the winning percentages was ignored and the graph showing a larger prize tended to be selected. In contrast, for a combination with a significant difference in the winning percentages such as a and e, then e, the choice with a higher winning percentage, tended to be preferred. This tendency illustrates a relation of $a \succ b$, $b \succ c$, $c \succ d$, $d \succ e$, $e \succ a$, which demonstrably does not satisfy transitivity.

Table 2.1 Experimental tasks examining transitivity-1

Gamble	Winning probability	Outcome (in $)	Expected value (in $)
a	7⁄24	5.00	1.46
b	8⁄24	4.75	1.58
c	9⁄24	4.50	1.69
d	10⁄24	4.25	1.77
e	11⁄24	4.00	1.83

Source: Tversky (1969). Reproduced in part by author

Tversky (1969) also presented the percentile rank points from the assessment of the intelligence, emotional stability, and sociability of five college applicants, as in Table 2.2, to test subjects. He had them answer which applicant should be admitted to a college based on a paired comparison with the priority on their intelligence. For a comparative judgment such as those between a and b and between b and c, a slight difference in the intelligence assessment was ignored and other factors with a better assessment tended to be selected. In contrast, for a combination with a significant difference in the intelligence assessment such as a and e, then e, with a better intelligence assessment, tended to be preferred. This result also indicates a relation of $a \succ b$, $b \succ c$, $c \succ d$, $d \succ e$, $e \succ a$, which clearly does not satisfy transitivity.

Table 2.2 Experimental tasks examining transitivity-2

Applicant	Intelligence	Emotional stability	Sociability
a	69	84	75
b	72	78	65
c	75	72	55
d	78	66	45
e	81	60	35

Source: Tversky (1969). Reproduced in part by author

Tversky (1969) proposed a mathematical model called the additive difference model to explain such preferences that do not satisfy the transitivity. This model first posits that a set of alternatives consists of multiple attributes $A = A_1 \times A_2 \times \ldots \times A_m$ as shown in Table 2.2. Additionally, each alternative is regarded as comprising the values of multiple attributes such as $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$. The additive difference model is expressed as follows using u_1 as a real-valued function and ϕ_i as an increasing function.

$$x \succsim y \Leftrightarrow \sum\nolimits_{i=1}^{n} \phi_i[u_i(x_i) - u_i(y_i)] \geq 0$$

where $\phi_i(-\delta_i) = -\phi_i(\delta_i)$, $\delta_i = u_i(x_i) - u_i(y_i)$ for an arbitrary attribute, i.

Assuming that $\phi_i(\delta_i) = t_i(\delta_i)$, $t_i > 0$, then

$$\sum\nolimits_{i=1}^{n} \phi_i[u_i(x_i) - u_i(y_i)]$$

$$= \sum\nolimits_{i=1}^{n} t_i u_i(x_i) - \sum\nolimits_{i=1}^{n} t_i u_i(y_i)$$

can be drawn. Furthermore, assuming that $v_i(x_i) = t_i u_i(x_i)$, Then the result is

$$x \succsim y \Leftrightarrow \sum\nolimits_{i=1}^{n} v_i(x_i) \geq \sum\nolimits_{i=1}^{n} v_i(y_i)$$

which produces an additive utility model. Although non-transitivity cannot be explained when ϕ_i can be assumed with such linearity, if ϕ_i is a step function with a threshold (e.g., if $\varepsilon \geq \delta$, then $\phi_i(\delta_i) = 0$ where ε is a threshold of ϕ_i), then this additive difference model can explain non-transitivity.

Nakamura (1992) conducted an experimental examination of the conditions that deviated from the transitivity. Results suggest the following: (1) In the case of a preference judgment based on a single attribute, the judgment was clarified, even if the difference in the utility was small. (2) When the utility of two or more attributes was traded off and the difference was slight, the judgment would be ambiguous. (3) If the utility of a certain attribute could be considered equivalent while the utility of other attributes might not be deemed equivalent, then the effect of the attribute that is probably equivalent would be neglected. In an effort to explain the non-transitivity of people's preferences, he proposed a preference model termed the additive fuzzy utility difference structure model, which assumes utility as a set with an ambiguous boundary: a fuzzy set.

The fact that the transitivity in a weak order would not always hold has been described. Based on experience, comparability is unlikely to hold at all times, as well. For example, inadequate knowledge of product brands would make it difficult to present a preference relation that would satisfy comparability in all cases. Furthermore, the study by Tversky (1969) forced test subjects to choose between two alternatives. In reality, however, selecting one from two might be difficult in some cases. Takemura (2007, 2012) expanded the model of Nakamura (1992) and the analytical techniques of Takemura (2000, 2005) and proposed a model that included the weight utility function and attempted to express, approximately, the preference relation that did not satisfy the transitivity.

3 Preference Reversal Phenomenon

Concepts similar to decision-making include the concept of judgment. Although decision-making is the act of selecting one from a group of alternatives, judgment is definable as the act of specifying the subject to a particular position in an assessment continuum. Judgment includes, for instance, assessing the risk of a traffic accident with a probability between zero and one and rating the desirability of a result on a scale of seven or nine levels of "not desirable at all" to "very desirable." Another example of judgment is pricing the value of alternatives in product brands. According to common reasoning, judgment and decision-making differ only in the patterns of reactions. They are expected to reflect preference and assessment in the same direction. As a result of judgment, for example, if the assessed value of an alternative x is found to be higher than that of an alternative y, then a relation by which x is likely to be chosen over y can be expected as a result of decision-making. A situation in which alternative y is likely to be chosen over alternative x in decision-making despite the higher assessed value of alternative x than that of alternative y in judgment is improbable based on common reasoning.

In general, utility theory implicitly assumes that the preference ranking relations of the assessed subjects are maintained even if many different methods are used. This assumption is apparently self-evident considering the measurement of the physical quantity. In other words, when weighing each of two fish that have been caught, for example, whether comparing their weights on a balance or measuring their weights in grams, there is expected to be little difference in the order relation as to which fish is heavier, even if the scales lack accuracy to some degree.

However, psychological studies conducted in the past suggest that preference order based on judgment and preference order based on decision-making are not necessarily the same and might be reversed in some cases. This phenomenon in which the preference order is reversed because of the difference between the reactions to judgment and decision-making is called the preference reversal phenomenon. This phenomenon was reported first by psychologists such as Lindman (1971) and Lichtenstein and Slovic (1971) as a phenomenon of preference relation inconsistency attributable to the methods of selection and pricing in gambles. The selection problem of these studies necessitated that test subjects choose between Gamble *H*, with a high winning percentage and a small amount of prize money (e.g., the winning percentage is 8/9 and a prize is $4) and Gamble *L*, with a low winning percentage and a large amount of prize money (e.g., the winning percentage 1/9 and a prize is $40). The pricing question asked the lowest probable prices at which Gamble *H* and Gamble *L* could be sold if the respondents owned them. In most cases, Gamble *H* was preferred in the selection problem and Gamble *L* was priced higher than the other in the pricing question (Tversky and Thaler 1990). Although many economists were skeptical about this preference reversal phenomenon initially, experimental economists repeatedly discovered effects that led to the recognition that this phenomenon certainly existed. The existence of this phenomenon implies that the type of reaction associated with judgment and decision-making affects the preference order rather than a phenomenon by which each result of judgment and decision-making is simply expressing a certain preference pattern (Tversky et al. 1988).

Tversky et al. (1988) examined other types of preference reversals in decision-making under certainty. In a study, the experimenter gave the following instructions to test subjects under each set of conditions.

Conditions for the selection problem: “In Israel, 600 people die in traffic accidents every year. The Ministry of Transport studied various measures to reduce the victims of traffic accidents. Please consider the following two proposed measures. The annual cost and the number of victims as a result of adopting each of the proposed measures are shown (Table 2.3). Which proposal would you adopt?”

Conditions for the matching problem: In the matching problem, a table resembling Table 2.4 with missing sections was presented to the test subjects, who were asked to deduce the missing information so that Proposed Measure X and Proposed Measure Y would become equivalent.

Table 2.3 Choice task

	Traffic fatalities	Cost
Program x	500 people	$55 million
Program y	570 people	$12 million

Source: Tversky et al. (1988). Reproduced in part by author

Table 2.4 Matching task

	Traffic fatalities	Cost
Program x	500 people	?
Program y	570 people	$12 million

Source: Tversky et al. (1988). Reproduced in part by author

Accepting the assumption that a decrease in the casualties in traffic accidents and a low cost of measures are desirable allows the prediction of the results of the selection problem based on the results of the matching problem. We assume, for example, that a test subject has estimated $40 million in the matching problem. Therefore, the profile of Proposed Measure X (500 victims and $40 million) and the profile of Proposed Measure Y (570 victims and $12 million) are equivalent. Based on the assumption, the profile of Proposed Measure X (500 victims and $40 million) and the profile of Proposed Measure Y (570 victims and $12 million) are superior to the profile of Proposed Measure X (500 victims and $55 million) in the selection problem. Based on such reasoning, Proposed Measure Y is predicted to be selected from the results of the matching problem.

Nonetheless, Tversky et al. (1988) found that most test subjects would select Proposed Measure X in the selection problem and that most would prefer Proposed Measure Y in the matching problem. Such a preference reversal phenomenon has been identified in personal and social decision-making processes and also in risk judgment and other situations (Grether and Plott 1979; Lindman 1971; Lichtenstein and Slovic 1971; Slovic 1995; Slovic et al. 1990; Starmer 2000; Tversky et al. 1988; Takemura 1994, 1996).

The following Chap. 3 will describe the causes of this preference reversal phenomenon and models to explain its occurrence.

References

Grether, D. M., & Plott, C. R. (1979). Economic theory and the preference reversal phenomenon. *American Economic Review, 69*, 623–638.

Kobashi, Y. (1988). *Ninchi Kagaku Sensho 18: Kettei wo Shien Suru [Selected works in cognitive science 18: Support decisions]*. Tokyo: University of Tokyo Press.

Krantz, D. H., Luce, R. D., Suppes, P., & Tversky, A. (1971). *Foundations of measurement Vol. 1: Additive and polynomial representations*. New York: Academic.

Lichtenstein, S. R., & Slovic, P. (1971). Reversals of preference between bids and choices in Gambling decisions. *Journal of Experimental Psychology, 89*, 46–55.

Lindman, H. R. (1971). Inconsistent preferences among Gambles. *Journal of Experimental Psychology, 89*, 390–397.

Nakamura, K. (1992). On the nature of intransitivity in human preferential judgments. In V. Novak, J. Ramik, M. Mares, M. Cherny, & J. Nekola (Eds.), *Fuzzy approach to reasoning and decision making* (pp. 147–162). Dordrecht: Kluwer.

Slovic, P. (1995). The construction of preferences. *American Psychologist, 50*, 346–371.

Slovic, P., Griffin, D., & Tversky, A. (1990). Compatibility effects in judgment and choice. In R. M. Hogarth (Ed.), *Insights in decision making: A tribute to Hillel J Einhorn* (pp. 5–27). Chicago: University of Chicago Press.

Starmer, C. (2000). Developments in non-expected utility theory: The hunt for descriptive theory of choice under risk. *Journal of Economic Literature, 38*, 332–382.

Takemura, K. (1994). Risuku handan ni okeru sentaku kadai to matchingu kadai no mujun [On the discrepancy of risk judgment in choice and matching tasks]. *Nihon risuku kenkyu gakkaishi [Japanese Journal of Risk Analysis], 5*(2), 88–95.

Takemura, K. (1996). Ishikettei to Sono Shien [Decision-making and support for decision-making]. In S. Ichikawa (Ed.), *Ninchi Shinrigaku 4kan Shikou [Cognitive psychology Vol. 4 thoughts]* (pp. 81–105). Tokyo: University of Tokyo Press.

Takemura, K. (2000). Vagueness in human judgment and decision making. In Z. Q. Liu & S. Miyamoto (Eds.), *Soft computing for human centered machines* (pp. 249–281). Tokyo: Springer.

Takemura, K. (2005). Fuzzy least squares regression analysis for social judgment study. *Journal of Advanced Intelligent Computing and Intelligent Informatics, 9*(5), 461–466.

Takemura, K. (2007). Ambiguous comparative judgment: Fuzzy set model and data analysis. *Japanese Psychological Research, 49*(2), 148–156.

Takemura, K. (2012). Ambiguity and social judgment: fuzzy set model and data analysis. In E. P. Dadios (Ed.), *Fuzzy logic-algorithms, techniques and implementations* (pp. 1–22). In Tech: Open Access Publisher

Tversky, A. (1969). Intransitivity of preferences. *Psychological Review, 76*, 31–48.

Tversky, A., Sattath, S., & Slovic, P. (1988). Contingent weighting in judgment and choice. *Psychological Review, 95*, 371–384.

Tversky, A., & Thaler, R. (1990). Anomalies: Preference reversals. *Journal of Economic Perspectives, 4*(2), 201–211.

Chapter 3
Causes of Preference Reversal Phenomenon

Chapter 2 provided descriptions of preference reversal phenomenon. This phenomenon was reported first by psychologists such as Lindman (1971) and Lichtenstein and Slovic (Slovic and Lichtenstein 1968; Lichtenstein and Slovic 1971) as the phenomenon of preference relation inconsistency that results from the methods of selection and pricing in gambles. The selection problem of these studies had the test subjects choose between Gamble *H* with a high winning percentage and a small amount of prize money (i.e., the winning percentage is 28/36 and the prize is $10) and Gamble *L* with a low winning percentage and a large amount of prize money (i.e., the winning percentage 3/36 and the prize is $100). The pricing question asked how much the lowest probable price at which Gamble *H* and Gamble *L* could be sold if the respondents owned them. In most cases, Gamble *H* was preferred in the selection problem and Gamble *L* was priced higher than the others in the pricing question (Slovic and Lichtenstein 1983; Slovic 1995; Tversky and Thaler 1990).

According to Tversky et al. (1990), this preference reversal phenomenon is divisible into three phases. The first is the phase of the discovery of this phenomenon by psychologists that occurred between the end of the 1960s and the beginning of the 1970s (e.g., Lindman 1971; Slovic and Lichtenstein 1968; Lichtenstein and Slovic 1971). The second was the phase of follow-up research conducted by economists between the end of the 1970s and the beginning of the 1980s, which led to the discovery that the preference reversal phenomenon was firmly present even if the monetary incentive was manipulated (e.g., Grether and Plott 1979). Subsequently, the third was the phase of theoretical studies designed to reveal the causes of the preference reversal phenomenon that began increasing in the 1980s (e.g., Loomes and Sugden 1989; Tversky et al. 1990). Studies designed to find the causes of the preference reversal phenomenon continue today in the fields of psychology and economics (Takemura 1996; Starmer 2000; Cubitt et al. 2004). This chapter introduces major studies that have sought the causes of the preference reversal phenomenon and characterizes this decision-making phenomenon.

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1 Summary of the Preference Reversal Phenomenon

Researchers have long argued about the manner in which the preference reversal phenomenon can be theoretically positioned and what the causes of this phenomenon are. These arguments are made through articles in economics magazines such as *Econometrica* and the *American Economic Review*. Various positions are adopted to interpret the preference reversal phenomenon. However, they is divisible broadly into an interpretation that this phenomenon can be described as a deviation from "transitivity," which assumes that no recurrence relation such as a three-cornered deadlock relation can occur in the preference relation, and an interpretation that transitivity is not necessary.

In an attempt to explain this, we will reorganize the preference reversal phenomenon and express it symbolically.

First, we assume that C_H and C_L are the monetary values (the lowest prices at which the seller is willing to sell) of Gambles H and L, respectively, that have been defined in advance. Subsequently, $\succ$ is assumed to be a strong preference relation. A strong preference relation is a relation by which something is preferred to others. It is such a relation by which, if the weak order relation, in which transitivity and comparability hold, is $\succsim$ and a set of alternatives is Set A, then $x \succsim y$ holds for the arbitrary elements, x and y, of Set A, but $y \succsim x$ does not hold. In other words, this relation is expressed in symbols as follows.

$$\forall x, y \in A, x \succsim y \wedge not(y \succsim x).$$

Furthermore, $\sim$ is assumed to be an indifference relation. An indifference relation is one in which something is liked about equally to some other thing. It is such a relation that if the weak order relation is $\succsim$, and the set of alternatives is Set A; then both $x \succsim y$ and $y \succsim x$ hold for the arbitrary elements x and y of Set A. In other words, this relation is expressed symbolically as

$$\forall x, y \in A, x \succsim y \wedge y \succsim x.$$

A standard preference reversal phenomenon represents a relation by which H is preferred to L more strongly (a strong preference relation holds) and C_L is greater than C_H. This relation can be expressed symbolically as shown below.

$$H \succ L \wedge C_L > C_H.$$

In addition, this case evidently assumes that a higher monetary value (higher price) is preferred. Consequently, it is assumed that if $X > Y$ holds for arbitrary monetary values, X and Y, $X \succ Y$ holds. Although a standard decision-making theory such as expected utility theory commonly used in economics assumes $H \succ L \wedge C_L > C_H$, the preference relation in the selection and pricing is reversed in the preference reversal phenomenon.

2 Explanation Based on Regret Theory That Assumes Non-transitivity

An example of major theories representing attempts to explain this preference reversal phenomenon using non-transitivity (Tversky 1969) is regret theory. This theory was proposed simultaneously by Bell (1982), Fishburn (1982), and Loomes and Sugden (1982). Among them, the studies that attempted to explain this theory by its application to the preference reversal phenomenon are Loomes and Sugden (1983, 1989). Their explanation (Loomes and Sugden 1989; Loomes et al. 1991) of the preference reversal phenomenon based on regret theory is described below.

First, they considered the case of decision-making under risk and defined the set of states as S and the state of n units of the elements of the set as $s_1, s_2, \ldots, s_n$ (Loomes and Sugden 1989). The results of each of the states when a certain alternative i was adopted was defined as $x = (x_{i1}, x_{i2}, \ldots, x_{in})$. Additionally, the probability of the state s_j to occur would be p_j, and $\sum_{j=1}^{n} p_j = 1$ was assumed.

The psychological assumption underlying regret theory is that, in a paired comparison of alternatives x_i and x_k, the decision-maker evaluates the alternatives not only from the results experienced in all states in S, but also from the relative relations with x_i and x_k.

The paired comparison model of alternatives x_i and x_k can be expressed as presented below.

$$x_i \succ x_k \Leftrightarrow \sum_{j=1}^{n} p_j \psi\left(x_{ij}, x_{kj}\right) > 0$$

$$x_i \sim x_k \Leftrightarrow \sum_{j=1}^{n} p_j \psi\left(x_{ij}, x_{kj}\right) = 0$$

$$x_i \prec x_k \Leftrightarrow \sum_{j=1}^{n} p_j \psi\left(x_{ij}, x_{kj}\right) < 0$$

Therein, $\psi(x_{ij}, x_{kj})$ represents the profit from selecting x_i over x_k with a real-valued function. Assuming that this represents skew-symmetry, then $\psi(x_{ij}, x_{kj}) = -\psi(x_{kj}, x_{ij})$ is inferred. Based on this, $\psi(x_{ij}, x_{ij}) = 0$ is found to hold for arbitrary x_{ij}.

Furthermore, they assumed the following characteristics, which indicate regret aversion as an attribute of this function ψ. Consequently, there are results, y_1, y_2, and y_3, of arbitrary money and the following relation holds when $y_3 > y_2 > y_1$.

$$\psi(y_3, y_1) > \psi(y_3, y_2) + \psi(y_2, y_1).$$

The greater the difference in the results becomes, the better the assessment. For that reason, the relation of this regret aversion expresses the urge of a person to avoid regretting that "the other alternative should have been selected" in advance.

On the premise of such an assumption of regret theory, the following explains the preference reversal phenomenon based on the composition of the alternatives presented in Table 3.1. The four lines, p_1, p_2, p_3, and p_4 in Table 3.1 represent the probability of each state to occur. The monetary result is assumed to be $a > b > c > d, e$. Alternative C (x_3 in Table 3.1) produces the same result in any of the state that occurs, constituting a definite alternative. If $p_1 + p_3 > 0.5 > p_1 + p_2$ then alternative L (x_3 in Table 3.1) can be interpreted as Gamble L with a low winning percentage and a large amount of prize money that is adopted in the preference reversal phenomenon, and alternative H (x_2 in Table 3.1) can be interpreted as Gamble H with a high winning percentage and a small amount of prize money.

Table 3.1 State, probability and outcomes in the decision making tasks

State	s_1	s_2	s_3	s_4
Probability	p_1	p_2	p_3	p_4
Alternative				
x_1(= Gamble L)	a	a	d	d
x_2(= Gamble H)	b	e	b	e
x_3(= Sure prize C)	c	c	c	c

Source: Loomes et al. (1989). Reproduced in part by author

According to Loomes and Sugden (1989), the preference reversal phenomenon represents non-transitivity that is $H \succ L, L \succ C$, and $C \succ H$. Therefore, the relation described below holds true based on Table 3.1 and the assumptions. In fact, they assumed that the preference relation would be a weak order even if the selection ranking was a strong preference relation and distinguished between the selection ranking and conjectured preference relations. This study, however, introduces both as equivalent to simplification of the explanation.

$$H \succ L \Leftrightarrow p_1\psi(b,a) + p_2\psi(e,a) + p_3\psi(b,d) + p_4\psi(e,d) > 0$$

$$L \succ C \Leftrightarrow p_1\psi(a,c) + p_2\psi(a,c) + p_3\psi(d,c) + p_4\psi(d,c) > 0$$

$$C \succ H \Leftrightarrow p_1\psi(c,b) + p_2\psi(c,e) + p_3\psi(c,b) + p_4\psi(c,e) > 0$$

Adding the assumption of skew-symmetry and summing and organizing the left side of the three equations above yield the following

$$\begin{aligned} p_1[\psi(a,c) - \psi(a,b) - \psi(b,c)] \\ - p_2[\psi(a,e) - \psi(a,c) - \psi(c,e)] \\ + p_3[\psi(b,d) - \psi(b,c) - \psi(c,d)] \\ + p_4[\psi(c,e) - \psi(c,d) - \psi(d,e)] \end{aligned}$$

Based on the assumption of regret aversion, the first three terms all become positive. The fourth item becomes non-negative if it is $d \geq e$ and negative if it is $d < e$.

Their regret theory explains that no state would contradict the preference reversal phenomenon (Loomes and Sugden 1989), and further assumes particularly that $p_2 = 0$ and $d \geq e$ would facilitate a simpler explanation of the preference reversal phenomenon. Table 3.1 apparently shows that such assumption would not change the qualitative attributes such as Gamble H, Gamble L, a definite alternative C. With this assumption in place, the first, third, and fourth terms are positive—only the second term is zero—making the value of the equation a positive number.

In this case, even if non-transitivity were to be established, non-transitivity patterns such as $L \succ H$, $H \succ C$, and $C \succ L$ could not occur from the assumptions of regret theory.

That earlier study presented various gambles that could be Gamble L and Gamble H in the preference reversal phenomenon shown in Table 3.2 and used them to conduct three experiments (Loomes and Sugden 1989). Experiment 1 used 283 subjects who were undergraduate and postgraduate students, Experiment 2 used 120 subjects, and Experiment 3 used 186 subjects. Experiment 1 found 29 subjects who indicated non-transitivity, of which 26 indicated non-transitivity, which was consistent with the predictions of regret theory. The other three displayed inconsistency with such predictions. Experiment 2 revealed 29 subjects demonstrating non-transitivity, of which 23 revealed non-transitivity consistent with the predictions of regret theory. The other six displayed inconsistency. In Experiment 3, half of the test subjects took selection problems only and the other half took a normal test of pricing. The presence of preference reversals can only be confirmed through normal tests. Of 93 subjects, 43 showed preference reversals, of which 28 revealed the preference reversal phenomenon that was in the predicted direction and 15 in the opposite direction. Non-transitivity was demonstrated by 18 subjects in the selection problems and by an estimated 17.75 subjects in the normal tests. Furthermore, 14 subjects were found to show behavior consistent with the predicted direction in the selection problems; an estimated 11.75 subjects were consistent in the normal tests. They demonstrated the validity of regret theory, maintaining that, of the non-transitivity patterns, the statistical tests had found a significant number of results predicted from regret theory. They additionally performed similar analysis in another experimental study (Loomes et al. 1991) and claimed the effectiveness of regret theory in explaining the preference reversal phenomenon.

Table 3.2 Gambles used in the experiment for the non-transitivity detection in the preference reversal phenomenon

Gamble L	Gamble H
$L_1 = \{£12.00, 0.4; £0.00, 0.6\}$	$H_1 = \{£8.00, 0.6; £0.00, 0.4\}$
$L_2 = \{£27.00, 0.2; -£1.00, 0.8\}$	$H_2 = \{£5.00, 0.8; -£1.00, 0.2\}$
$L_3 = \{£7.50, 0.4; £2.50, 0.6\}$	$H_3 = \{£6.00, 0.8; -£1.00, 0.2\}$
$L_4 = \{£8.00, 0.3; £3.00, 0.7\}$	$H_4 = \{£6.00, 0.7; £1.00, 0.3\}$
$L_5 = \{£9.00, 0.4; £1.50, 0.6\}$	$H_5 = \{£6.00, 0.75; £0.00, 0.25\}$

Note: {£ 12.00, 0.4; £ 0.00, 0.6} indicates a gamble for £ 12.00 with probability 0.4 and nothing with probability 0.6

Source: Loomes et al. (1989). Reproduced in part by author

3 Explanation Based on a "Deviation from Procedural Invariance" That Does Not Assume Non-transitivity

Although the regret theory of Loomes and Sugden explained the preference reversal phenomenon using the non-transitivity of preference relations, others have argued that non-transitivity is not necessary for the preference reversal phenomenon. This point is asserted particularly by a research group of psychologists including Tversky et al. (1990). Tversky et al. have claimed that although the explanation based on regret theory was certainly consistent with the direction of non-transitivity indicated by the test subjects, the subjects who displayed non-transitivity were few to begin with, comprising merely 10 % of the subjects who displayed preference reversals in the results of the analysis of the experiments conducted by them. The other 90 % did not demonstrate non-transitivity. They also pointed out that the test subjects indicating non-transitivity in the experiments of Loomes et al. comprised only 15–20 % of all subjects and argued that the preference reversal phenomenon could be more appropriately interpreted as rather than non-transitivity, a phenomenon that deviated from "procedural invariance," in which the preference relation was presumably maintained even if different procedures of preference revelation were used.

Tversky et al. claimed that the following relation holds in the standard preference reversal phenomenon.

$$H \succ L \wedge C_L > X > C_H.$$

Therein, X is the amount of money between C_L and C_H. They also stated that the following four preference patterns might logically exist without the assumption of equally ranked subjects.

1. Non-transitivity: A case in which $L \succ X$ and $X \succ H$ hold and result in $L \succ X \succ H \succ L$.
2. Overpricing of L: A case in which $X \succ H$ and $X \succ L$ hold and result in $C_L \succ X \succ L$.
3. Underpricing of H: A case in which $H \succ X$ and $L \succ X$ hold and result in $H \succ X \succ C_H$.
4. Concurrence of overpricing of L and underpricing of H: A case in which $H \succ X$ and $X \succ L$ hold and result in $H \succ X \succ C_H$ and $C_L \succ X \succ L$.

They performed a preference reversal experiment on 198 men and women who had been gathered through an advertisement on college bulletins for estimating these conceivable patterns (Experiment 1). All 620 reactions in the experiment were categorized into the four patterns described above, which is presented in Table 3.3. The results reveal that the number of cases that were explainable with non-transitivity is small (10.0 %), many of which resulted from overpricing of Gamble L with a low winning percentage and a large amount of prize money (only the overpricing of L is 65.5 %, which totals 83.9 % when combined with the concurrence of underpricing of H). These results suggest also that the preference reversal phenomenon should be understood as a deviation from procedural invariance rather than as non-transitivity.

Table 3.3 Pattern of preference reversal

Pattern	N	Percent (%)	Explanation
$L > X, X > H$	62	10.0	Non-transitivity
$X > L, X > H$	406	65.5	Overweight to L
$L > X, H > X$	38	6.1	Underweight to H
$H > X, X > L$	114	18.4	Overweight to L and underweight to H

Souce: Tversky et al. (1990). Reproduced in part by author

The preference reversal phenomenon is explainable even without the assumption of non-transitivity of preference when viewed from a deviation from an axiom of expected utility theory called the independence axiom in utility theory (Holt 1986; Karni and Safra 1987) or a deviation from the reduction axiom (Segal 1988). It can also be explained consistently based on nonlinear utility theory, which is an extension of expected utility theory. According to the opinions related to these positions, the preference reversal phenomenon is explainable consistently simply by eliminating the independence axiom or reduction axiom from expected utility theory even without the procedural invariance assumed by Tversky et al. (1990) or non-transitivity assumed by Loomes and Sugden (1989).

If $x \succ y$ holds for arbitrary results $x, y, z \in A$, then the independence axiom means $px(1 - p)z \succ py + (1 - p)z$ for arbitrary $p \in [0,1]$. In other words, assuming that the independence axiom is appropriate, this represents that the decision-maker’s entire preference relation depends only on preference relations with x and y and is independent from the probability of obtaining z that commonly exists. Additionally, the reduction axiom is an axiom that demands that a compound lottery or compound gamble be reduced to a simple lottery or gamble to be treated. In other words, in the reduction axiom, if Lottery or Gamble G, from which Result x is obtainable with Probability p and Result y is obtainable with Probability $1 - p$, are further combined and Gamble G is obtainable with Probability q and Result y is obtainable with Probability $1 - q$, such a compound lottery or compound gamble is demanded to be indifferent from a simple lottery or simple gamble, from which Result x is gained with Probability pq and Result y is achieved with Probability $1 - pq$.

Researchers who attempt to explain the preference reversal phenomenon using deviations from such independence axiom or reduction axiom have not conducted particular experiments, but have merely presented theoretical possibilities. In the experiment results of Tversky et al. (1990), however, 90 % of the data showed a deviation from procedural invariance. They argued that neither a deviation from the independence axiom nor a deviation from the reduction axiom would constitute the necessary or sufficient conditions to explain the preference reversal phenomenon.

Consequently, Tversky et al. (1990) states that the preference reversal phenomenon can be interpreted as a deviation from procedural invariance, which results from overpricing of Gamble *L*. Why, then, is Gamble *L* overvalued in the pricing? The next chapter presents a description of the psychological interpretation of this phenomenon and the contingent-weighting model (Tversky et al. 1988; Slovic et al. 1990; Takemura 1994) that supports the interpretation and introduces some decision-making phenomena related to this psychological process.

References

Bell, D. E. (1982). Regret in decision making under uncertainty. *Operations Research, 39*, 961–981.

Cubitt, R. P., Munro, A., & Starmer, C. (2004). Testing explanations of preference reversal. *Economic Journal, 114*, 709–726.

Fishburn, P. C. (1982). Nontransitive measurable utility. *Journal of Mathematical Psychology, 26*, 31–67.

Grether, D. M., & Plott, C. R. (1979). Economic theory and the preference reversal phenomenon. *American Economic Review, 69*, 623–638.

Holt, C. A. (1986). Preference reversals and the independence axiom. *American Economic Review, 76*, 508–515.

Karni, E., & Safra, Z. (1987). 'Preference reversal' and the observability of preferences by experimental Methods. *Econometrica, 55*, 675–685.

Lichtenstein, S. R., & Slovic, P. (1971). Reversals of preference between bids and choices in Gambling decisions. *Journal of Experimental Psychology, 89*, 46–55.

Lindman, H. R. (1971). Inconsistent preferences among Gambles. *Journal of Experimental Psychology, 89*, 390–397.

Loomes, G., Starmer, C., & Sugden, R. (1989). Preference reversal: Information-processing effect or rational non-transitive choice? *The Economic Journal, 99*, 146.

Loomes, G., Starmer, C., & Sugden, R. (1991). Observing violations of transitivity by experimental methods. *Econometrica, 59*, 425–439.

Loomes, G., & Sugden, R. (1982). Regret theory: An alternative theory of rational choice under uncertainty. *Economic Journal, 92*, 805–824.

Loomes, G., & Sugden, R. (1983). Rationale for preference reversal. *American Economic Review, 73*, 428–432.

Loomes, G., & Sugden, R. (1989). Preference reversal: Information-processing effect or rational transitive choice? *Economic Journal, 99*, 140–151.

Segal, U. (1988). Does the preference reversal phenomenon necessarily contradict the independence axiom? *American Economic Review, 78*, 233–236.

Slovic, P. (1995). The construction of preferences. *American Psychologist, 50*, 346–371.

Slovic, P., Griffin, D., & Tversky, A. (1990). Compatibility effects in judgment and choice. In R. M. Hogarth (Ed.), *Insights in decision making: A tribute to Hillel J Einhorn* (pp. 5–27). Chicago: University of Chicago Press.

Slovic, P., & Lichtenstein, S. (1968). The relative importance of probabilities and payoffs in risk taking. *Journal of Experimental Psychology Monograph Supplement, 78*, 1–18.

Slovic, P., & Lichtenstein, S. (1983). Preference reversals: A broader perspective. *American Economic Review, 73*, 596–605.

Starmer, C. (2000). Developments in non-expected utility theory: The hunt for descriptive theory of choice under risk. *Journal of Economic Literature, 38*, 332–382.

Takemura, K. (1994). Risuku handan ni okeru sentaku kadai to matchingu kadai no mujun [On the discrepancy of risk judgment in choice and matching tasks]. *Nihon risuku kenkyu gakkaishi [Japanese Journal of Risk Analysis], 5*(2), 88–95.

Takemura, K. (1996). Ishikettei to Sono Shien [Decision-making and support for decision-making]. In S. Ichikawa (Ed.), *Ninchi Shinrigaku 4kan Shikou [Cognitive psychology vol. 4 thoughts]* (pp. 81–105). Tokyo: University of Tokyo Press.

Tversky, A. (1969). Intransitivity of preferences. *Psychological Review, 76*, 31–48.

Tversky, A., Sattath, S., & Slovic, P. (1988). Contingent weighting in judgment and choice. *Psychological Review, 95*, 371–384.

Tversky, A., Slovic, P., & Kahneman, D. (1990). The cause of preference reversal. *American Economic Review, 80*, 204–217.

Tversky, A., & Thaler, R. (1990). Anomalies: preference reversals. *Journal of Economic Perspectives, 4*(2), 201–211.

Chapter 4
Psychology of Preference Reversals and Prominence Hypothesis

Chapter 3 introduced the experimental research of Tversky et al. (1990), arguing that the preference reversal phenomenon can be interpreted as a deviation from procedural invariance. A deviation from procedural invariance refers to a trait by which preference is reversed by preference revelation procedures. Tversky et al. asserted that the preference reversal phenomenon was explainable by partial modification of expected utility theory such as transitivity and independence axiom. This chapter will describe the prominence hypothesis as a psychological interpretation of this phenomenon and the contingent-weighting model (Tversky et al. 1988; Slovic et al. 1990) of the specific representation of the hypothesis and introduce some experiments related to the model.

1 Prominence Hypothesis and Preference Reversal Phenomenon

Tversky et al. (1988) developed the contingent weighting model to explain the preference-reversal phenomenon. This model was built on the assumption of the prominence hypothesis. The prominence hypothesis assumes that attributes that are more noticeable are weighted in selection problems rather than pricing problems, and more lexicographic decisions are made.

Lexicographic decision-making refers to a method of evaluating alternatives in which the alternatives with the most desirable attribute values among the attributes that are emphasized the most are the most highly regarded. If some attributes are equally ranked among the most examined attributes, they are compared similarly among the next most examined attributes. According to their contingent weighting model, the decision-maker does not determine the preference order in a strict sense lexicographically (i.e., not the lexicographic order in a strict sense). Rather, the model includes the assumption that decisions are made in a form resembling the

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_4,

lexicographic order in which prominent attributes are weighted more heavily in the selection problems.

In an attempt to explain this prominence hypothesis, we assume a situation in which a college graduate selects her place of employment following the examination of Tversky et al. (1988).

Kaori, a college student, has decided after applying for jobs that she will choose either Company A or Company B as her employer. She will make the decision based on two attributes: the appeal of the work and the amount of the salary. If Company A surpasses Company B in both the appeal of the work and the amount of salary, then she could easily choose Company A because Company A is superior in all attributes to be considered. If, however, Company A were to pay higher salaries and if Company B were to offer a more appealing job, then neither option would be superior to the other.

In such a case, she confronts a tradeoff between the attributes. A tradeoff between the attributes means, in essence, that if the job is fulfilling, she must tolerate the lower salary to some degree. The tradeoff places a psychological burden on her because it involves some tolerance, possible sacrifice, and consideration of various aspects of the problem. Then, the decision-maker is likely to reconstruct decision-making problems psychologically or reconsider particular attributes to create alternatives that are psychologically superior. If, for example, she prioritizes the appeal of work over the amount of salary, then the appeal of work is likely to be emphasized more and the salary will be less important in the selection problem according to the prominence hypothesis.

As this example illustrates, the psychological process by which people are expected to distort the way they understand decision-making problems or consider biased attributes was assumed not only in the prominence hypothesis of Tversky et al. (1988), but also in the dominance structure search model of Montgomery (1983, 1993). These two, however, mutually differ in the sense that, although the dominance structure search model concerns the psychological process of decision-making, the prominence hypothesis is to predict the relative weight between the attributes in decision-making.

According to Tversky et al. (1988), the following two can be presented as psychological reasons for the weight of more prominent attributes in decision-making in selection problems to grow heavier and the decision-making method to become lexicographic, as represented by the prominence hypothesis. The first is that a lexicographic method of decision-making requires no tradeoff between the attributes: decision-making is possible without making great mental and emotional effort. In other words, this gives a benefit on the part of the decision-maker with little burden of data processing required for lexicographic method of decision-making (in short, decision-making will become easier). The other reason is that the lexicographic method of decision-making facilitates the type of decision-making that involves a tradeoff between attributes justifying the reasons why the selected alternative is superior to the other alternatives that are not selected. Put succinctly, because fewer attributes are examined, the reason for the superiority is easy to explain to both the decision-maker himself and others. For these reasons, Tversky et al. inferred that the prominence hypothesis would hold true in the preference reversal phenomenon.

In pricing problems, the weight of decision-making on prominent attributes is smaller than in the selection problems, and the prominence hypothesis assumes that the lexicographic method of decision-making is unlikely to be adopted. What is the psychological reason for this? In pricing problems, the winning percentages of two gambles are known. Some adjustment is made so that the differences between the two winning percentages and the amounts of prize money become equal. At such a point, the weight of the winning percentages and amounts of prize money are thought to be adjusted after the amounts of prize money because a mooring point is determined such that the differences in the winning percentages and amounts of prize money would be equal. Studies of judgment in the past have discovered that when adjusting the assessed value after the mooring point is determined, the adjustment is incomplete and an assessed value that has not changed much from the mooring point tends to be estimated (e.g., Kahneman et al. 1982). For this reason, the relative weight of prominent attributes was predicted to be smaller in pricing problems than in selection problems.

2 Contingent Weighting Model

Tversky et al. (1988) developed the contingent weighting model to explain the preference reversal phenomenon based on the prominence hypothesis. This model is an approximate expression of the prominence hypothesis. The prominence hypothesis can explain not only the inconsistency in preference in the selection problems and pricing problems in the preference reversal phenomenon, but also the inconsistency in the overall matching problems such as selection problems and pricing. Therefore, this model based on this prominence hypothesis is also useful to explain the inconsistency phenomenon in the entire selection and matching problems in addition to the preference reversal phenomenon. Accordingly, the following will expand the pricing problems to the matching problems and explain them.

According to the expression of Tversky et al. (1988), this model takes the following format.

The set of the first and most examined attributes is $A = \{a, b, c, \ldots\}$; the set of the secondary attributes to be examined next is $Z = \{x, y, z, \ldots\}$. Then we consider the subject of selection as a Cartesian product: $A \times Z$. The preference relations $\succsim_c$ and $\succsim_m$ that satisfy a weak order are assumed to be a preference relation obtained through a selection and a preference relation obtained through matching, respectively.

We assume that the value of one attribute is independent from the fixed value of the other attribute. In other words, for arbitrary $a, b \in A$ and $y, z \in Z$,

$$ay \succsim_i by \Leftrightarrow az \succsim_i bz$$
$$bz \succsim_i by \Leftrightarrow az \succsim_i ay$$

hold where $i = c, m$.

Based on the assumption presented above, we assume that functions F_i and G_i that indicate the following relations that are defined on A and Z, respectively, exist. In other words, for arbitrary $a, b \in A$ and $y, z \in Z$,

$$az \succsim_i by \Leftrightarrow F_i(a) + G_i(z) \ \geq \ F_i(b) + G_i(y) \tag{4.1}$$

holds where $i = c, m$.

We also consider that $F_i^{'}$ and $G_i^{'}$ are derived functions of F_i and G_i for $i = c, m$ and assume $RS_i = F_i^{'}/G_i^{'}$ and that the ratio between RS_c and RS_m is constant at each point. In other words, for arbitrary $a \in A$ and $z \in Z$,

$$\frac{RX_c(a,z)}{RS_m(a,z)} = \text{constant} \tag{4.2}$$

By assuming (4.1) and (4.2), the existence of functions F and G that indicate the following relations that are defined on A and Z, respectively, is presented. In other words, for arbitrary $a, b \in A$ and $y, z \in Z$,

$$\begin{aligned} & az \succsim_i by \\ & \Leftrightarrow \alpha_i F_i(a) + \beta_i G_i(z) \ \geq \ \alpha_i F_i(b) + \beta_i G_i(y) \\ & \Leftrightarrow F_i(a) + \theta_i G_i(z) \ \geq \ F_i(b) + \theta_i G_i(y) \end{aligned} \tag{4.3}$$

hold where $\theta_i = \alpha_i/\beta_i$ and $i = c, m$.

Tversky et al. (1988) attempted to explain the preference reversals in the selection and matching problems based on the relations presented in (4.3). Therefore, they expected that preference reversals would occur because of differences in the values of θ_c and θ_m. Under the conditions that (4.3) holds true, the indifference curves of the attributes receiving the most emphasis and the attributes receiving the second most emphasis are parallel straight lines in the areas of selection problems and matching problems, as portrayed in Fig. 4.1. If $\theta = \theta_c/\theta_m$, then the case of $\theta = 1$ represents a case in which preference reversals are not occurring, and the case of $\theta < 1$ represents a case in which the prominence hypothesis holds true.

Tversky et al. (1988) proposed the contingent weighting model, which assumes that such weight, θ, varies depending on the selection problem. Assuming for simplification that F and G are linear functions, then relation (4.3) becomes the following.

$$\begin{aligned} az \succsim_i by \ &\Leftrightarrow \ \alpha_i a + \beta_i z \ \geq \ \alpha_i b + \beta_i y \\ &\Leftrightarrow \ a + \theta_i z \ \geq \ b + \theta_i y \end{aligned} \tag{4.4}$$

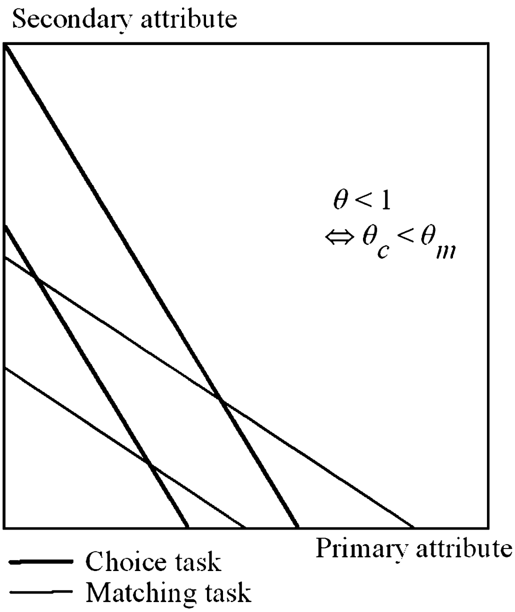


Fig. 4.1 Indifference curves for attributes in a contingent weighting model. *Source*: Tversky et al. (1988). Reproduced in part by author

We also assume that $C(az, by)$ and $M(az, by)$ respectively represent the ratios of preferring by to az in a selection problem and a matching problem. Preference reversals might occur in selection problems and matching problems. Therefore, $M(az, by) = C(az, by)$ does not generally hold true. Accordingly, if y^* is a value that satisfies $M(az, by^*) = C(az, by)$, then this holds when the following relation can exist based on (4.4) and the additional assumption of stochastic conjoint measurement (Tversky et al. 1988).

$$\theta_m(z - y^*) = \theta_c(z - y) \tag{4.5}$$

Based on this,

$$\theta = \frac{\theta_c}{\theta_m} = \frac{(z - y^*)}{(z - y)}$$

is defined.

According to the prominence hypothesis, the value of θ is less than 1. This hypothesis predicts that the weight of prominent attributes is greater in selection problems than in matching problems.

3 Verification Experiments for Contingent Weighting Model

The following describes how the verification experiments for the contingent weighting model are performed and analyzed based on the study of Takemura (1994). This study used psychological experiments to examine whether the contradiction in the judgment based on the selection problems and matching problems developed by Tversky et al. (1988) were observable in the risk judgment of the identical test subject in an extremely urgent situation. This study included six experiment topics, of which the first three were related to social decision-making issues associated with risk; the last three used judgment issues relevant to the scale of risk.

The subjects in this experiment were 23 adults, of whom 21 were junior and senior engineering students at the University of Tsukuba. The other two were high-school teachers. The test subjects received instructions on the topic in a classroom and worked on selection and matching problems. The experimenter explained the topic while writing the problems on the blackboard.

1. Selection problem

   The topic was a decision-making problem, as presented below.

   Every year, many people who lose their lives in traffic accidents. Some measures to reduce traffic accidents have been developed according to the government's policy, and eventually, the following two proposals were presented. Please look at the table on the blackboard (Table 4.1). Proposed Measure x is predicted to reduce deaths in traffic accidents by 85 %, but it costs ¥1,000 million (approximately $10 million). Proposed Measure Y is predicted to reduce deaths in traffic accidents by 38 %, but it costs only ¥200 million (approximately $2 million). If you were to decide on the policy, which proposal would you adopt?
2. Matching problem

   After completing the selection problem, the experimenter gave the following instructions related to the matching problem.

   Please look at the table on the blackboard (Table 4.2). The cost of Proposed Measure x is missing. Please estimate the cost so that Proposed Measures X and Y will be equivalent.

The preference rates of the proposed measures in the selection and matching problems are presented in Table 4.3. The test subjects who had reversed their preference (judgment contradiction) on an individual basis were seven (30.4 %) of those who changed their preference from X to Y and none from Y to X. In the matching problem, the median of the estimated values of the missing value was ¥600 million (approximately $6 million). Those test subjects who indicated a contradiction of risk assessment between the selection and matching problems were seven individuals (30.4 %). The 95 % confidence intervals of the rates based on angular transformation (Iwahara 1964) were 8.3–59.2 %. This result implies that the minimum of 8.3 % to the maximum of 59.2 % of the test subjects are estimated to be cause contradiction in their risk assessment at the 95 % confidence interval. Such results suggest the occurrence of preference reversals between the selection and matching problems and the change from the tendency to examine reducing mortality specifically in the selection problem to the tendency to examine the cost in the matching problem specifically at both the individual and group levels. Like Tversky et al. (1988), assuming that the first attribute to be emphasized the most was the reduction of the percentage of deaths in traffic accidents and that the second attribute was the cost, then the θ value was 0.50. This result is consistent with the prominence hypothesis because the θ value is less than 1 in this hypothesis.

Table 4.1 Choice task

	Traffic casualities	Cost
Program *X*	85 % decreased	¥1,000 million
Program *Y*	38 % decreased	¥200 million

Source: Takemura (1994)

Table 4.2 Matching task

	Traffic casualities	Cost
Program *X*	85 % decreased	?
Program *Y*	38 % decreased	¥200 million

Source: Takemura (1994)

Table 4.3 Results of choice task and matching task

	Choice task (%)	Matching task (%)
Program *X*	12 (52.2)	4 (17.4)
Program *Y*	11 (47.8)	18 (78.3)
Indifferent	0 (0.0)	1 (4.3)

Source: Takemura (1994)

This study revealed a substantial contradiction of judgment ranging from 30–60 % in the first three experiments on social decision-making related to risk. The judgment contradiction among 10–20 % of the test subjects in the last three experiments on the scale of risk. The reasons for the difference in the percentages of test subjects who demonstrated contradiction is likely to include the difference in the characteristics of the judgment problems. The issues for which the selection aspects are emphasized, such as the case of social decision-making, appear more likely to cause risk judgment contradiction than assessment issues such as the scale of risk. Despite such differences in the issues, this study revealed that only a change in the measurement procedure in risk judgment could cause preference results to be reversed completely by the same test subject, even in an extremely urgent situation.

The θ values based on the results of this experiment were 0.50, 0.40, and 1.20 in the experiments of social decision-making and 0.83, 0.79, and 0.84 in experiments in the judgment of the scale of risk. With the exception of results of the decision-making experiment on automobile safety devices, the range of values consistent with the hypothesis of Tversky et al. (1988) was obtained. Although the fact that the value θ was greater than 1 in only one instance leaves room for future examination, the series of experiment results can be regarded as corresponding generally to the prominence hypothesis that

- in selection problems, more important attributes such as mortality are emphasized and the specific examination of secondary attributes is reduced; and
- conversely, in matching problems, the weight of the second attributes including cost increases, thereby showing the effect that corresponds to the prominence hypothesis.

4 Interpretation of Interpret Procedural Invariance: The Scale Compatibility Principle

The prominence hypothesis presented herein indicates a deviation from procedural invariance, in which the psychological process of decision-making varies depending on the preference revelation procedures, and explains the preference reversal phenomenon and contradiction between the selection and matching problems as a result of such deviation. Why, then, is the prominence hypothesis developed? Tversky et al. (1988, 1990) and Slovic et al. (1990) proposed the scale compatibility principle as a more general principle underlying the prominence hypothesis.

The scale compatibility principle states that the weight of attributes in the assessment of a subject increases if the type and measure of the response and assessed attributes mutually correspond. In the preference reversal phenomenon, for instance, the weight of the amount of prize money is expected to increase in the pricing problem, which is a measure of response to an amount of money. Conversely, in the selection problem, a decision with weight on the winning percentage, which is considered important in the selection, is expected to be made. In fact, we know that gambles with a low winning percentage and a large amount of prize money are highly regarded in the preference reversal phenomenon. This scale compatibility principle consistently explains such preference patterns in the preference reversal phenomenon.

In addition, the following can be predicted from the scale compatibility principle in connection with the preference reversal phenomenon. In the matching problem, pricing with some prize money information missing makes the weight of the amount of prize money higher than when estimating the winning percentage with inadequate winning percentage data. Consequently, a gamble with a low winning percentage and large amount of prize money (Gamble L) will be preferred to a gamble with a high winning percentage and small amount of prize money (Gamble *H*). Slovic et al. (1990) conducted an experiment with 200 students at the University of Oregon to examine this hypothesis. The results are presented in Table 4.4, which reveals that the results are consistent with the hypothesis based on the scale compatibility principle.

Table 4.4 Percentage of responses favoring the H bet over the L bet for four different tasks

	Choice task	Probability matching	Matching task	Pricing
Small bets (H, L)				
(35⁄36, $4), (11⁄36, $16)	80	79	54	29
(29⁄36, $2), (7⁄36, $9)	75	62	44	26
(34⁄36, $3), (18⁄36, $6.50)	73	76	70	39
(32⁄36, $4), (4⁄36, $40)	69	70	26	42
(34⁄36, $2.50), (14⁄36, $8.50)	71	80	43	22
(33⁄36, $2), (18⁄36, $5)	56	66	69	18
Mean	71	72	50	29
Large bets (H, L)				
(35⁄36, $100), (11⁄36, $400)	88	76	69	65
(29⁄36, $50), (7⁄36, $225)	83	64	31	55
(34⁄36, $75), (18⁄36, $160)	77	79	65	55
(32⁄36, $100), (4⁄36, $1,000)	84	68	28	61
(34⁄36, $65), (14⁄36, $210)	78	80	36	57
(33⁄36, $50), (18⁄36, $125)	68	75	58	46
Mean	80	74	48	56
Overall mean	76	73	49	37

Source: Slovic et al. (1990). Reproduced in part by author

Finally, the phenomenon called deviation from procedural invariance explained in this chapter appears to conflict with the premise of psychology and social science that one true value exists in the assessed values. Currently, few reasonable grounds exist for whether it is the selection problem or matching problem that can measure the true value of the judgment-maker. Addressing the issues such as which method should be used to make social judgments that are accepted by more people or what types of methods are justifiable is necessary in the future. Furthermore, future judgments and decision-making studies are necessary to consider this apparent contradiction in human judgment.

References

Iwahara, S. (1964). *Non parametric ho [Non-parametric methods]*. Tokyo: Nihon Bunka Kagakusha.

Kahneman, D., Slovic, P., & Tversky, A. (Eds.). (1982). *Judgment under uncertainty: Heuristics and biases*. Cambridge: Cambridge University Press.

Montgomery, H. (1983). Decision rules and the search for a dominance structure: Towards a process model of decision making. In P. C. Humphreys, O. Svenson, & A. Vari (Eds.), *Analyzing and aiding decision processes* (pp. 343–369). Amsterdam: North-Holland.

Montgomery, H. (1993). The search for a dominance structure in decision making: Examining the evidence. In G. A. Klein, J. Orasanu, R. Calderwood, & C. E. Zsambok (Eds.), *Decision making in action: Models and methods* (pp. 182–187). New Jersey: Ablex.

Slovic, P., Griffin, D., & Tversky, A. (1990). Compatibility effects in judgment and choice. In R. M. Hogarth (Ed.), *Insights in decision making: A tribute to Hillel J Einhorn* (pp. 5–27). Chicago: University of Chicago Press.

Takemura, K. (1994). Risuku handan ni okeru sentaku kadai to matchingu kadai no mujun [On the discrepancy of risk judgment in choice and matching tasks]. *Nihon risuku kenkyu gakkaishi [Japanese Journal of Risk Analysis], 5*(2), 88–95.

Tversky, A., Sattath, S., & Slovic, P. (1988). Contingent weighting in judgment and choice. *Psychological Review, 95*, 371–384.

Tversky, A., Slovic, P., & Kahneman, D. (1990). The cause of preference reversal. *American Economic Review, 80*, 204–217.

Part III
Expected Utility Theory and Its Counterexamples

Chapter 5
Expected Utility Theory and Psychology

Expected utility theory is a major theory of decision making under risk. Decision making under risk is a type of decision-making in which the probability distribution of the results is known. This expected utility theory is assumed in numerous theories of economics. Certainly, any student taking economics should have been taught it in a course of some kind.

The history of expected utility theory is long; it had already been proposed in the eighteenth century and is closely related to psychology constituting the origin of behavioral decision theory. This chapter first explains the basic idea of the initial expected utility theory and then, and how this theory is related to psychological theories. Subsequently, psychological studies of utility measurement based on expected utility theory will be introduced in the final part.

1 The St. Petersburg Paradox and Expected Utility

Decision making under risk is often explained using the idea of expected value of utility, namely, expected utility. For example, the utility of taking an umbrella when going out can be described as follows:

$$\begin{aligned} &EU(\textit{taking an umbrella when going out}) \\ &\quad = p_1(\textit{It rains}) \times u_1(\textit{going out with an umbrella when it rains}) \\ &\qquad + p_2(\textit{It does not rain}) \times u_2(\textit{going out with an umbrella when it does not rain}) \end{aligned}$$

In that equation, p_1 and p_2 are probabilities and $p_1 + p_2 = 1$ based on the probability axiom. Such a theory that addresses the expected value of utility in decision-making under risk is called expected utility theory. The type that assumes a subjective probability is particularly called subjective expected utility theory.

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_5,

1.1 St. Petersburg Paradox

Expected utility theory of decision making under risk dates back to the formulation of D. Bernoulli, an eighteenth century mathematician. He proposed the application of expected utility to solve the St. Petersburg paradox introduced by his uncle, N. Bernoulli.

This paradox can be described as follows (Tamura et al. 1997). A coin with the probability of showing a head or tail being 0.5 is tossed repeatedly until a head appears. Win $\$2^n$ if it is tossed n times until the first head appears. The question, then, is what the maximum price considered fair to pay to participate in the game would be (Apparently, the original unit of money is replaced with dollar here).

Assuming that the trials are repeated infinite times, the expected value EV of participating in this game would be

$$EV = \sum_{n=1}^{\infty} 2^n \cdot 2^{-n} = 2\left(\frac{1}{2}\right) + 4\left(\frac{1}{4}\right) + 8\left(\frac{1}{8}\right) + \cdots = 1 + 1 + 1 + \cdots = \infty$$

which would exceed any finite price if the expected value is the judgment criterion.

The author administered a questionnaire survey to undergraduate and graduate classes on the question of up to how much would be the acceptable price to pay for this game. Most students responded that they would pay less than \$10. This answer was generally common among humanities students at Waseda University and science and technology students at University of Tsukuba and Tokyo Institute of Technology.

Such intuition of people contradicts the idea of expected value. People's intuition should not have varied significantly since the time of Bernoulli, which is presumably why it was called a paradox.

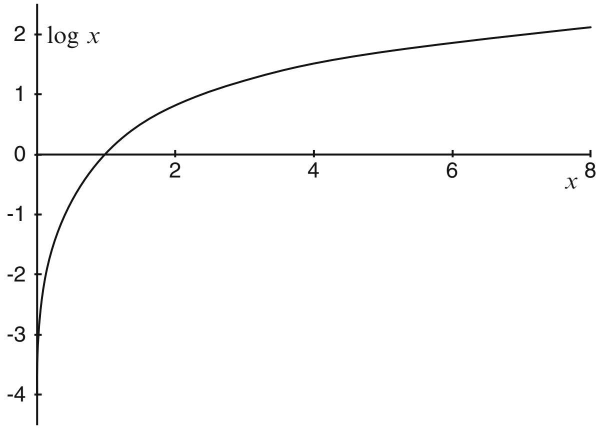
1.2 Solution to the Paradox

Bernoulli considered the following expected utility, EU, which is the expected value of the utility of logarithmic function $u(2^n) = \log(2^n)$.

$$EU = \sum_{n=1}^{\infty} \log(2^n) \cdot (2^{-n}) = \log 4$$

He demonstrated that the expected utility in this example would converge to a rather low finite value, i.e., log4 (approximately 1.4) which is worth \$4. He argued that the paradox would be solved by considering such an expected value of the logarithmic utility function.

Fig. 5.1 Logarithmic utility function



The utility function that is expressed with such a logarithmic function shown in Fig. 5.1 indicates a characteristic of "diminishing marginal utility": the larger the amount of money, the lower the rate of increase in utility. In actuality, that is a function that is concave downward (concave function), signifying risk-averse decision-making.

This paradox, however, might not be a paradox if we assume that the budget of the banker is finite as suggested by Kaneko (2003). As exemplified by Kaneko (2003), even if the banker's budget were equivalent to \$80 trillion (more than 20 times of a national budget of the United States and more than 70 times of a national budget of Japan) is only between the 46th and 47th power of \$2. Therefore, the expected value of the game is only between \$46 and \$47, which translates into the following.

$$\sum_{n=1}^{46} 2^n \cdot 2^{-n} = 46 < EV < \sum_{n=1}^{47} 2^n \cdot 2^{-n} = 47$$

Therefore, even if the banker had as much money as the national budget of the United States, the expected value would be considerably low and would not be a paradox.

Although some might argue that the interpretation of St. Petersburg paradox need not consider the logarithmic utility function that was formulated by Bernoulli, the inclusion of this function can be fully justified when considered from a psychological perspective. The following describes the relevance between the logarithmic utility function and research findings in the perceived quantities in psychology.

2 Relevance Between Fechner's Psychophysics and Logarithmic Utility Function

Since early times, the field of psychophysics has existed in psychology, in which the theory of perceived quantity that can be expressed in a logarithmic function has been developed. The perceived quantity is defined based on human sensory judgment and utility is defined based on selection and preference, whose measurement procedures therefore mutually differ despite their remarkably similar concepts.

The founder of psychophysics, G. T. Fechner, proposed psychophysical methods of measurement in his book, *Elemente der Psychophysik* (*Elements of Psychophysics*) published in 1860 (Fechner, 1860). He developed the constant measurement method and scaling method to identify the functional relation between stimulus intensity and psychological quantity gained through judgment and derived the theory of perceived quantity that is expressed in the logarithmic function.

He proposed the theory, the so-called Fechner's law, that the intensity of a sensation, S, as judged based on the experiment findings of E. H. Weber et al. that the ratio, $\Delta I/I$, of stimulus intensity, I, and its differential threshold, ΔI, is constant, which is the so-called Weber's law, and is proportional to the logarithm of the stimulus intensity, I ($S = k\log I$ where k is a constant) (Wada et al. 1969; Indo 1977).

Weber's law states that an increase in a barely detectable stimulus, i.e., the differential threshold, is proportional to the initial intensity of the stimulus. The following experiment, might be a helpful illustration.

A test subject holds two glasses filled with water (both mass whose value is I) as in Fig. 5.2 and closes his eyes. Then, the other person slowly pours water into either of the glasses. The test subject tells the other person which one has become heavier when he feels the additional weight. Then, the increase (ΔI) in the mass of the water is measured.

Through these steps, the value of the Weber ratio, $\Delta I/I$, can be inferred. This experiment is repeated multiple times for each person to calculate the average Weber ratio. Weber's law holds that when the glass with water weighs 100 g, even if the additional weight is sensed when more water is added to produce 115 g in comparison to 100 g, the difference between 200 and 215 g cannot be felt, but the difference between 200 and 230 g is recognizable. In other words, Weber's law holds that the ratio, $\Delta I/I$, between the stimulus intensity, I, and its differential threshold, ΔI, is constant.

This law is known to be applicable to various senses such as hearing, vision, and touch (Wada et al. 1969). Not only it is applicable to such basic senses, but it is also known to be generally valid for such a sense as the feeling of having struck a good bargain when given a product price discount (Kojima 1986). Weber's law suggests, for example, that a 30-cents discount on a $1 product and the same discount on a

Fig. 5.2 Experiment of comparison judgment

$100 product would not give the same sense of a bargain, although a 30-cents discount on a $1 product and a $30 discount on $100 product would be felt equally as a good bargain.

In the derivation of Fechner's law, he considered ΔI in differentiation, assumed $\Delta I = dI$, and further assumed that this was proportional to the minimum unit $\Delta S = dS$ of senses, leading to $dS = k\ dI/I$ (k is a constant). He took the integral of both sides of this equation to form

$$S = k\log I + C\ (C \text{ is a constant}).$$

If the stimulus intensity is I_0 when $S = 0$, then $C = -\ k\log I_0$ should result, which therefore engenders

$$S = k\log I - k\log I_0 = k\log \frac{I}{I_0}.$$

If we assume that I/I_0 is a stimulus intensity that has been standardized by the stimulus threshold value I_0, we obtain the so-called Fechner's law, which can be expressed using the same formula as that of Bernoulli's logarithmic utility function. Assuming the perceived amount of money, a diminishing effect that is equivalent to the nature of diminishing marginal utility in the utility function assumed by Bernoulli is visible.

3 Possible Psychophysical Laws and Utility Function

Gustav Theodor Fechner

Born in 1801; deceased in 1887. According to Schultz (1981), Fechner began studying medicine at the age of 16 at the University of Leipzig in 1817. He started to give lectures at that university in 1824, and became a professor at the university in 1833. His studies cover widely various fields, including physiology, physics, mathematics, psychology, experimental aesthetics, and philosophy (Schultz, 1981).

He is the founder of psychophysics and demonstrated that psychology could be studied using quantitative models and experiments. His psychophysical studies not only strongly affected modern psychology, but also left a substantial impact on such philosophers as E. Mach, E. Husserl, and H. Bergson.

Photograph: Aflo

Like this Fechner's law, the law for the relation between physical and psychological quantities is called a psychophysical law, on which various studies have been conducted up to the present. The propriety of the psychophysical function in Fechner's logarithmic function has been challenged by some who criticize that the derivation of Weber's law involves an illogical leap, and by S. S. Stevens who argues in his theory ($S = \alpha I^{\beta}$ where α and β are constants) that a power function, rather than a logarithmic function, is appropriate (Wada et al. 1969). Although such arguments are made, Fechner's logarithmic function, together with the power function of Stevens (1975), is generally accepted as a psychophysical function for stimuli and reactions.

In connection with this, D. Luce—a mathematical psychologist—used functional equations to develop an argument about "possible psychophysical laws" that correspond to an interval scale and a ratio scale (Luce 1959, 1990). His study was an attempt to derive theoretically possible psychophysical laws from the perspective of an admissible transformation of scales.

First, assuming that perceived quantity $u(I)$ is an interval scale, then changing the unit of the scale value of I and converting it by constant multiplication (k times) can be expected to yield the following functional equation because $u(I)$ is thought be linearly transformed based on the definition of an interval scale, which means that

$$v(kI) = K(k)u(I) + C(k)\ k > 0,\ K(k) > 0.$$

Luce has proved that the only continuous function, $u(I)$, that could satisfy the above functional equation above would be the following two functions.

$$u(I) = \alpha \log I + \beta$$
$$u(I) = \alpha I^{\beta}$$

Therefore, Fechner's law or Stevens' law holds when the judgment uses an interval scale.

If the stimulus is I ratio scale and the perceived quantity, then $v(I)$ is also a ratio scale based on the definition of a ratio scale; changing the unit of the scale value of stimulus I and converting it by constant multiplication (k times) is only likely to multiply the scale $v(I)(> 0)$ of the perceived quantity by $K(k)$ in response, resulting in homomorphic correspondence. Therefore, the following functional equation is thought to hold.

$$v(kI) = K(k)v(I)\ k > 0,\ K(k) > 0.$$

Luce proved that the continuous function $v(I)$ that satisfies the functional equation above would be the following power function.

$$v(I) = \alpha I^{\beta}, \alpha > 0$$

Consequently, Luce's theoretical study described here can be summarized as showing that when the stimulus can be measured using a ratio scale, Stevens' law holds if human judgment uses a ratio scale and Fechner's law or Stevens' holds if the judgment uses an interval scale. Although neither law is related to a utility function, it is interesting that Fechner's law resembles Bernoulli's logarithmic utility function and that Stevens' utility function resembles the Cobb–Douglas function, which is often used in economics.

This Luce's formulation has been criticized by some who argue that it is not applicable to a case in which there is no unit of stimulus intensity (i.e., being made dimensionless) (Indo 1977). Researchers are still discussing Luce's formulation to the present day (Iverson and Luce 1998). Additionally, Takemura (1998, 2001) proposed an assessment function for the judgment that consumers make. The mental ruler theory alleges the function's characteristics as being concave downward near the lower limit of a stimulus that can be judged and convex downward near the upper limit. That formulation includes the laws of Fechner and Stevens as special cases.

As suggested, the conclusion of Luce's formulation necessitates further studies. Despite such arguments, Fechner's logarithmic function and Stevens' power function are generally accepted to a considerable degree as a psychophysical function for stimuli and reactions. Apart from the field of senses, numerous theories that use value functions and utility functions equivalent to the psychophysical functions of Fechner and Stevens have been developed in the theories of value and utility.

For example, estimation of value function using a power function is conducted also in nonlinear utility theories such as the prospect theory of A. Tversky and D. Kahneman, which describes the assessment of monetary benefits (Tversky and Kahneman 1992). Because the measurement procedures for the utility function and psychophysical function mutually differ, even if they are expressed in the same formula, their axiomatic systems differ. Therefore, they cannot be considered identical. However, they certainly share a close relation.

Amos Tversky

Born in 1937; deceased in 1996. Graduated from the Hebrew University of Jerusalem in 1961 and earned a Ph.D. degree from University of Michigan in 1965. He taught classes at the Hebrew University of Jerusalem between 1966 and 1978 and at Stanford University from 1978 until he passed away.

He spent many years studying decision-making under risk and uncertainty with Daniel Kahneman, the winner of the 2002 Nobel Prize in economics. There must be many researchers who think that he would have received the Nobel Prize jointly with Kahneman if he had lived until 2002.

He also conducted joint research on the preference reversal phenomenon with Paul Slovic, who is well-known for his studies of risk perception. Although he is considered a pioneer of behavioral economics, he is highly recognized among psychologists in the fields of cognitive psychology and cognitive social psychology that include cognitive biases.

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He visited Japan in 1992 and gave seminars at a symposium of the Japanese Psychological Association, Tokyo Institute of Technology, Doshisha University, and the University of Tsukuba.

Ed Souza/Stanford News Service

4 Study of the Measurement of Utility Based on the Expected Utility Theory

Von Neumann and Morgenstern (1944/1947) demonstrated that a utility function based on an objective probability exists when multiple axioms are satisfied; that utility can be measured when the expected utility theory is assumed. Unlike Bernoulli's expected utility theory, their expected utility theory does not necessarily assume a logarithmic utility function, but they formulate a utility function in a more abstract form.

The expected utility in the expected utility theory of von Neumann and Morgenstern is expressed as follows (Tamura et al. 1997). First, we assume that the set of alternatives is

$$A = \{a_l, a_m, \ldots\}$$

and that the probability of obtaining the result x_i when the decision-maker selects the alternative $a_l \in A$ is p_i, the probability of obtaining the result x_i when $a_m \in A$ is selected is q_i, …, and the set of all possible results is $X = \{x_1, x_2, \ldots\}$. At this point, we assume $p_i \geq 0$, $q_i \geq 0$, … for all of i, and

$$\sum_i p_i = \sum_i q_i = \cdots = 1$$

is satisfied. When the utility function on X is $u : X \rightarrow R$, then the expected utility when the alternatives, a_l, a_m, ... are adopted is

$$Ea_l = \sum_i p_i u(x_i)$$

$$Ea_m = \sum_i q_i u(x_i)$$

respectively. This expected utility theory assumes that the decision-maker adopts the alternative that results in the maximum expected utility from the set, A, of alternatives. In addition, this utility function is known not to lose its essential meaning even after a positive linear transformation and is known to have the nature of cardinal utility (interval scale).

Mosteller and Nogee (1951) conducted an experiment to measure people's utility after assuming the expected utility theory. They explained the gamble by expressing it as a payoff matrix shown in Table 5.1 to the test subjects and repeatedly asking them whether they would accept the gamble of poker dice in an attempt to measure utility. In their experiment, the probabilities of all outcomes in the poker dice are presented to the test subjects.

Table 5.1 Pay-off matrix for utility measurement-1

	Win	Lose
Gambling	x	−5
No gambling	0	0

Source: Mosteller and Nogee (1951). Reproduced in part by author

According to the expected utility theory, the equation below holds true when the expected utility values of gambling and not gambling are equivalent:

$$u(0) = pu(x) + (1 - p)u(-5).$$

Therein, p is the known probability of winning the gamble. Because the utility is cardinal utility and unique up to a positive linear transformation, arbitrarily assuming $u(0) = 0$, $u(-5) = -1$ and solving $u(x)$ result in

$$u(x) = \frac{1 - p}{p}.$$

They attempted to measure the utility by considering the payoff (x) systematically and setting the condition in which the probabilities of selecting gambling and not gambling are both 50 % as the equivalence point.

Nine Harvard University students and five National Guard members of the State of Massachusetts participated as test subjects; numerous trials were conducted over a period of 4 months. Utility ranging from \$0.05 to \$5.50, depending on the experiment, was measured. Consequently, a concave downward utility function was obtained for the students and a downward convex utility function was acquired for the National Guard members.

A problem with the experiment of Mosteller and Nogee (1951) was that, although the objective probability was presented to the test subjects in their experiment, they were unable to eliminate the possibility that the probability was converted to a subjective figure. Particularly based on the idea of the subjective expected utility theory of Savage (1954), their experiment was unable to identify whether the subjective probability had changed or whether the utility had changed. According to the subjective expected utility theory of Savage, if multiple axioms of selection behavior were accepted, then it would be equivalent to the fact that people's preference relations select the alternative that would maximize the subjective expected utility based on subjective probability.

To overcome such a problem, Davidson et al. (1957) assumed subjective expected utility theory and considered an experiment in which the subjective probability would become 1/2 in an attempt to measure both the utility and subjective probability. If the probability of event E is 1/2, then the subjective probability of its complementary event E^c also becomes 1/2 according to the probability axioms. Under subjective expected utility theory, if a gamble in which x is gained when E occurs and y is gained when E^c occurs, and conversely, a gamble in which y is gained when E occurs and x is gained when E^c occurs are not different, then the subjective probability of E and that of E^c are equal and 1/2.

Because test subjects tend to prefer either heads or tails systematically in coin tossing, they used a six-sided die with meaningless words such as "ZEJ" written on three sides and "ZOJ" written on the other three sides to eliminate the systematic preferences and created an event, E, that would generate a subjective equal probability. Subsequently, they performed a gamble selection experiment by changing the payoffs in the payoff matrix, as shown in Table 5.2.

Table 5.2 Pay-off matrix for utility measurement-2

	E	E^c
Gamble G_1	x	y
Gamble G_2	x	w

Source: Davidson et al. (1957). Reproduced in part by author

According to subjective expected utility theory, the utility function, u, and subjective probability, s, that satisfy the following relation exist.

$$G_1 \gtrsim G_2 \Leftrightarrow s(E)u(x) + s(E^c)u(y) \geq s(E)u(z) + s(E^c)u(w).$$

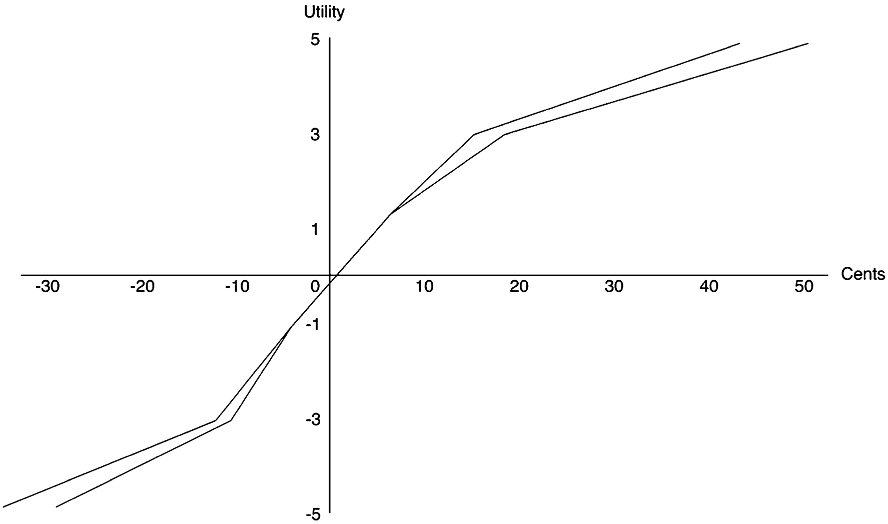

Fig. 5.3 Estimated utility function. *Source*: Davidson et al. (1957). Reproduced in part by author

Furthermore, because they set E so that $s(E) = s(E^c)$ in their experiment, the subjective probability is eliminated, resulting in

$$G_1 \succsim G_2 \Leftrightarrow u(x) + u(y) \geq u(z) + u(w).$$

By arbitrarily selecting the payoffs, they arbitrarily set the upper and lower limits of the utility function at 5 and $-$ 5, respectively, from the inequality derived from the selection. They subsequently estimated the utility value and inferred the form of the utility function (see Fig. 5.3).

This experiment estimated the utility of 15 test subjects selected by removing 4 who had made a decision contradictory to the theory from among 19 subjects at Stanford University.

They also arbitrarily selected the payoffs so that G_1 and G_2 would be mutually indifferent and assumed

$$\begin{aligned} G_1 &\sim G_2 \\ &\Leftrightarrow s(F)u(x) + s(F^c)u(y) \\ &= s(F)u(z) + s(F^c)u(w) \end{aligned}$$

based on subjective expected utility theory, which engenders $s(F) + s(F^c) = 1$ Therefore,

$$s(F) = \frac{u(w) - u(y)}{u(x) - u(z) + u(w) - u(y)}$$

and concluded that the subjective probability $s(F)$ would be determined.

Using this procedure, they measured the subjective probability of an event whose objective probability was 1/4, discovering that most of the test subjects tended to underestimate the objective probability.

References

Davidson, D., Suppes, P., & Siegel, S. (1957). *Decision making: An experimental approach*. Stanford: Stanford University Press.

Fechner, G. (1860). *Elemente der psychophysik*. Leibzing: Breitkopf & Hartel.

Indo, T. (Ed.). (1977). *Shinri sokutei, gakusyu riron [Psychological measurement, learning theory]*. Tokyo: Morikita Publishing.

Iverson, G., & Luce, R. D. (1998). The representational measurement approach to problems. In M. H. Birnbaum (Ed.), *Measurement, judgment, and decision making* (pp. 1–79). San Diego: Academic.

Kaneko, M. (2003). *Game riron to konnyaku mondo [Game theory and incoherent questions and answers]*. Tokyo: Nippon Hyoron sha.

Kojima, S. (1986). *Kakaku no shinri: Syohisya ha nani wo konyu kettei no monosashi ni surunoka [Price psychology: What measure do consumers use to make purchasing decisions?]*. Tokyo: Diamond Inc.

Luce, R. D. (1959). On the possible psychophysical laws. *Psychological Review, 66*, 81–95.

Luce, R. D. (1990). On the possible psychophysical laws, revisited: Remarks on cross-modal matching. *Psychological Review, 97*, 66–77.

Mosteller, F., & Nogee, P. (1951). An experimental measurement of utility. *Journal of Political Economy, 59*, 371–404.

Savage, L. J. (1954). *The foundations of statistics*. New York: Wiley.

Schultz, D. (1981). *A History of modern psychology [Murata, K. (Trans.) (1986). Gendai shinrigaku no rekishi. Tokyo: Baifukan]* (3rd ed.). New York: Academic.

Stevens, S. S. (1975). *Psychophysics: Introduction to its perceptual, neural and social prospects*. New York: Wiley.

Takemura, K. (1998). Jokyo izonteki isikettei no teiseiteki moderu: Shinteki monosashi riron niyoru setsumei [Qualitative model of contingent decision-making: An explanation of using the mental ruler theory]. *Ninchi Kagaku [Cognitive Studies], 5*(4), 17–34.

Takemura, K. (2001). Contingent decision making in the social world. In C. M. Allwood & M. Selart (Eds.), *Decision making: Social and creative dimensions* (pp. 153–173). Dordrecht: Kluwer Academic.

Tamura, H., Nakamura, Y., & Fujita, S. (1997). *Koyo bunseki no suri to oyo [Mathematical principles and application of utility analysis]*. Tokyo: Corona Publishing.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5*, 297–323.

von Neumann, J., & Morgenstern, O. (1944/1947). *Theory and games and economic behavior*. Princeton: Princeton University Press.

Wada, Y., Oyama, T., & Imai, S. (Eds.). (1969). *Kankaku + chikaku shinrigaku handobukku [A handbook for sensory and perception psychology]*. Tokyo: Seishin Shobo.

Chapter 6
Axioms and Counterexamples Expected Utility Theory

Chapter 5 explained the relevance between the initial idea of expected utility theory and psychology and introduced some studies of utility measurement based on expected utility theory. This chapter will first explain the axiomatic system of expected utility theory, then how to approach the axiomatic system, with introduction of some counterexamples.

A study of one theory of decision-making that derives a group of axioms expressing decision-making and preference relations (Barberà et al. 1998; Bell et al. 1988; Edwards 1992; Fishburn 1988; Ichikawa 1983; Iverson and Luce 1998; von Neumann and Morgenstern 1944/1947; Savage 1954; Tamura et al. 1997). This approach is an axiomatic method that has been adopted by mathematical psychologists and mathematical economists, which is intended to develop a structure of theoretical studies to derive the small number of qualitative axioms that underpin the quantitative models of decision-making. Empirical testing of the axioms of decision theory supports the investigation of the essential characteristics of decision-making and preference relations. A group of axioms has been studied empirically in the field of behavioral decision theory.

Empirical examination of the axioms of expected utility theory has produced some findings that cannot be fully supported: the Allais paradox (Allais 1953) and the Ellsberg paradox (Ellsberg 1961).

1 Decision-Making Under Risk and Premises of Expected Utility Theory

1.1 Review of the Structure of Decision-Making Under Risk

Before explaining the axioms of expected utility theory, we will review the structure of decision-making under risk. First, when a set of finite alternatives is A and its elements are organized as mutually exclusive alternatives $a_1, \ldots, a_i, \ldots, a_l$

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_6,

(l is the number of alternatives), then the set can be written as $A = \{a_1, \ldots, a_i, \ldots, a_l\}$. Subsequently, we consider the set, $X = \{x_1, \ldots, x_j, \ldots, x_m\}$, which is the result of adopting these alternatives. For instance, the elements of X include x_1 = a gain of \$100, x_2 = no gains, and x_3 = a gain of \$200. When a specific alternative, a_i, is adopted, a result, x_j, is likely to appear. However, a_i and x_j are not necessarily mutually correspondent. Result x_j of adopting alternative a_i should depend at least on a state of some kind, $\Theta = \{\theta_1, \ldots, \theta_k, \ldots, \theta_n\}$, and the probability distribution of Θ in decision-making under risk is known.

We assume, for example, some gambles in which a die is rolled, yielding the following results.

$$\theta_1 = 1, 2, or\ 3,$$
$$\theta_2 = 4, or\ 5, or$$
$$\theta_3 = 6.$$

We also assume that the amount of prize money is determined depending on the pips on the dice rolled, as shown in Table 6.1.

Table 6.1 Examples of outcomes corresponding to alternative states

	θ		
A	θ_1: 1, 2, 3	θ_2: 4, 5	θ_3: 6
a_1: Gamble 1	x_1: \$100	x_2: \$0	x_1: \$100
a_2: Gamble 2	x_1: \$100	x_2: \$0	x_3: \$200
a_3: Gamble 3	x_3: \$200	x_3: \$200	x_1: \$100

Table 6.1 reveals that the result is determined by the function (mapping) from the alternative selected and state to the result, which is

$$f : A \times \Theta \rightarrow X,$$

where

$$A \times \Theta = \{(a_i, \theta_k) | a_i \in A, \theta_k \in \Theta\}.$$

Regarding the probability in this case, the probability of θ_1 is $p(\theta_1) = 1/2$, the probability of θ_2 is $p(\theta_2) = 1/3$, and the probability of θ_3 is $p(\theta_3) = 1/6$. These probabilities can also be considered in terms of subjective probability based on frequency theory. Consequently, the probability on the results, X, for each alternative $a_j \in A$ can be determined as presented in Table 6.2. For instance, p_{33} in Table 6.2 is the probability of the result (X_3) that \$200 is gained when Gamble 3 (a_3) is selected.

According to Table 6.1, this is the result of the states θ_1 and θ_2; the probability p_{33} becomes $p(\theta_1) + p(\theta_2) = 1/2 + 1/3 = 5/6$, leading to $p_{33} = 5/6$ shown in Table 6.2.

Table 6.2 Examples of probability distributions of outcomes in decision making under risk

	X		
A	x_1: \$100	x_2: \$0	x_3: \$200
a_1: Gamble 1	p_{11}: 2/3	p_{12}: 1/3	p_{13}: 0
a_2: Gamble 2	p_{21}: 1/2	p_{22}: 1/3	p_{23}: 1/6
a_3: Gamble 3	p_{31}: 1/6	p_{32}: 0	p_{33}: 5/6

Therefore, the question of decision-making under risk by which one alternative, $a_i \in A$, should be selected can be replaced with the question of which of the following probability distributions on X should be selected.

$$\begin{array}{l} p_1 = [p_{11}, p_{12}, \ldots, p_{1m}] \\ p_2 = [p_{21}, p_{22}, \ldots, p_{2m}] \\ \ldots \\ p_l = [p_{l1}, p_{l2}, \ldots, p_{lm}] \end{array}$$

These equations signify that decision-making under risk can be expressed with a preference structure $\langle P, \succ \rangle$, in which a preference relation, $\succ$, is added to the set of probabilities $P = \{p_1, p_2, \ldots, p_l\}$ on X (This was explained in Chap. 1 using a different example).

1.2 Redefinition of Gambling

In an attempt to examine decision-making under risk further, we will first review the definition of probability by following the explanation of Tamura et al. (1997) to redefine gambling.

First, we consider the set of results, X. The A subset, $E(E \subset X)$, of this set X is an element of 2^X of the power set of $X(E \in 2^X)$. In this case, the power set of X refers to the set of all subsets of the set X, which is expressed as 2^X. Each element of a power set is a set itself. When $X = \{x_1, x_2, x_3\}$, for instance, 2^X is a set consisting of the following eight elements (ϕ, however, is an empty set).

$$2^X = \{\phi, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}\}.$$

We next consider what is called a finitely additive probability measure, p, on 2^X. The name, finitely additive probability measure might sound forbidding, but in simple terms, this refers to such a “probability” that can be expressed, for instance, as $p(\{x_1\}) = 0.2$. The finitely additive probability measure, p, on 2^X is a set function that satisfies the following conditions for all $E_i, E_j \in 2^X$.

1. $p(X) = 1$
2. $p(E_i) \geq 0$
3. $E_i \cap E_j = \phi \Rightarrow p(E_i \cup E_j) = p(E_i) + p(E_j)$

In other words, if (1) the probability of the entire set of results, X, is 1, then (2) the probability of an arbitrary subset, E_i of X is 0, and (3) the product set, $E_i \cap E_j$, of an arbitrary subset of X is an empty set (meaning that E_i and E_j do not intersect mutually), the probability of the union of E_i and E_j (i.e., the set combining E_i and E_j) is equivalent to $p(E_i) + p(E_j)$.

Next, we consider a convex set, P_X, of the finitely additive probability measure on 2^X (which is simply designated as a probability measure in the following). When P_X is a convex set, if $1 \geq \lambda \geq 0$ and arbitrary p and q are elements of $P_X(p, q \in P_X)$, then $\lambda p + (1 - \lambda)q$ is also an element of $P_X(\lambda p + (1 - \lambda) q \in P_X)$. In other words, a combination of the probabilities of two arbitrary results still represents an element of P_X.

When $E_i \in P_X$ is a finite set, the probability measure resulting in $p(E_i) = 1$ is said to be simple. This simple probability measure can be interpreted as representing gambling and lotteries based on the example of Table 6.2. Therefore, if P_X is a convex set, then this can be interpreted that a compound lottery or compound gambling that combines multiple lotteries or gambles with certain probabilities, λ and $1 - \lambda$, is also an element of P_X.

2 Axiomatic System of Expected Utility Theory

Ernst H. Weber

Born in 1795; deceased in 1878. Born in Wittenberg, Germany, he earned a doctorate from the University of Leipzig in 1875 and taught anatomy and physiology at the same university between 1817 and 1871 (Schultz 1981). His primary research themes concerned the physiology of sensory organs, which has strongly influenced modern psychology. He invented the method of measuring the concept of threshold and studied the minimum distinguishable value of the distance between two points on skin. He also established Weber's law that the minimum distinguishable difference between the amounts of physical stimulus is approximately proportional to the amount of physical stimulus. Weber's law holds not only in the area of senses such as tactile sense, vision, hearing, but it has been known to hold approximately also in

(continued)

(continued)

price judgment including the feeling of good bargain when given a product price discount.

We continue to explain the axiomatic system of expected utility theory using the expressions of Tamura et al. (1997).

First, because P_X can be interpreted as a set of alternatives, we consider the binary relations on P_X and can assume a real valued function, Φ, on $P_X \times P_X$ that satisfies $p \succ q \Leftrightarrow \Phi(p,q) > 0$ for all $p, q \in P_X$. Therein, $\succ$ represents a strong preference relation (i.e., $\forall\, p, q \in P_X,\ p \succsim q \wedge not(q \succsim p)$), where $\succsim$ is a weak preference relation).

Based on this real-valued function, Φ, the expected utility theory of von Neumann and Morgenstern (1944/1947) introduced in the previous chapter is explained using the following linear utility model.

2.1 Linear Utility Model

A linear utility model refers to a linear functional, U, on P_X that satisfies $\Phi(p, q) = U(p) - U(q)$ for all $p, q \in P_X$. A linear function is definable as follows: assuming that P_X is a linear space on R, when mapping $U : P_X \rightarrow R$ has the following two properties (linearity), i.e., when the following are true,

1. $\forall\, p, q \in P_X,\ U(p + q) = U(p) + U(q)$
2. $\forall\, a \in R, \forall\, p \in P_X,\ U(ap) = aU(p)$

then U is a linear functional on P_X.

The fact that U is linear means, in a different expression, that

$U(\lambda p + (1 - \lambda)q) = \lambda U(p) + (1 - \lambda)U(q)$ is true for all $p, q \in P_X$ and all $0 < \lambda < 1$.

Therein, Φ maintains its uniqueness even after positive constant multiplication (meaning it is a ratio scale) based on the definition of the linearity of U. Therefore, we know that U maintains its uniqueness with a positive linear transformation (meaning it is an interval scale). The reason is that the assumption of $U' = \alpha U + \beta$ $(\alpha > 0)$ results in $\alpha\Phi(p,q) = U'(p) - U'(q)$.

A linear utility model based on the utility $U(p_i)$ of a simple probability measure, p_i, that generates m units of results $x_j \in X$ of Gamble $a_i \in A$ with the probability of $p_{ij}(\sum_{j=1}^{m} p_{ij} = 1)$ each can be regarded as seeking the expected value of $U(x_j)$ because $U(p_i) = \sum_{j=1}^{m} p_{ij}U(x_j)$ holds based on the linearity of U, in which $U(p_i)$ is seeking the expected value of $U(x_j)$. In this sense, this linear utility model, U, can be regarded as an expected utility model. It also means that the expected utility theory of von Neumann and Morgenstern (1944/1947) seeks expected utility using the linear utility model U.

Some necessary and sufficient conditions exist for the expected utility theory of von Neumann and Morgenstern (1944/1947) to hold. Although they also presented an axiomatic system exhibiting necessary and sufficient conditions, in general, the axiomatic system of Jensen (1967) is often used, as provided below. We assume that the axiomatic system displayed below holds true for all $p, q \in P_X$ and all $0 < \lambda < 1$ that were defined earlier (the expressions of axiomatic systems are based on Tamura et al. 1997).

Axiom A1 (Order Axiom) $\succ$ on P_x represents a weak order.

The fact that the preference relation $\succ$ is a weak order means that

1. asymmetry, $p \succ q \Rightarrow not(q \succ p)$, and
2. negative transitivity, $not(p \succ q) \wedge not(q \succ r) \Rightarrow not(p \succ r)$,

hold true.

This is equivalent to the fact that

1. transitivity, $p \succsim q \wedge q \succsim r \Rightarrow p \succsim r$, and
2. comparability, $\forall\, p, q \in P_x, p \succsim q \vee q \succsim p$

hold for the weak preference relation, $\succsim$.

Axiom A2 (independence axiom) If $p \succ q$, then $\lambda p + (1 - \lambda)r \succ \lambda q + (1 - \lambda)r$.

Axiom A3 (continuity axiom) If $p \succ q$ and $q \succ r$, then certain $\alpha, \beta \in (0,1)$ exists and $\alpha p + (1 - \alpha)r \succ q$ and $q \succ \beta p + (1 - \beta)r$.

2.2 Theorem of Expected Utility of von Neumann and Morgenstern

When and only when Axioms A1, A2, and A3 hold, the linear functional, U, on P_x exists and

$$p \succ q \Rightarrow U(p) > U(q)$$

holds for all p, $q \in P_x$. Additionally, U maintains its uniqueness with a positive linear transformation (U is an interval scale).

Axiom A2, the independence axiom, is a necessary and sufficient condition because U is linear, and Axiom A3, the continuity axiom, is a necessary axiom because U is a mapping to the set, P_x, of real numbers.

The independence axiom is a particularly important property in expected utility theory. Deviation from this axiom can be interpreted as causing the Allais paradox and Ellsberg paradox that will be described later. The independence axiom means that when the preference relation of two alternatives (gamble) is certain, even if another gamble with equivalent results and probability of each result is combined, the preference relation of these alternatives is maintained.

We assume, for instance, that Gamble 2 is preferred to Gamble 1 in the case of the gamble in Table 6.2. Composing compound gambles incorporating the pair of Gamble 1 and Gamble 3 and the pair of Gamble 2 and Gamble 3 with a probability of 0.5 results in Gamble 1 and Gamble 2 in Table 6.3. The independence axiom demands that Gamble 2' be preferred to Gamble 1' if Gamble 2 is preferred to Gamble 1.

Table 6.3 Examples of compound gambles

	X		
A	x_1: \$100	x_2: \$0	x_3: \$200
a_1: Gamble 1	p_{11}: 5/12	p_{12}: 1/6	p_{13}: 5/12
a_2: Gamble 2	p_{21}: 1/3	p_{22}: 1/6	p_{23}: 1/2

3 Counterexamples of Expected Utility Theory

Does such utility theory reflect the decision-making of actual people? The phenomena called the Allais paradox (see Fig. 6.1) and the Ellsberg paradox (see Fig. 6.2) constitute counterexamples of expected utility theory, which deviate from the independence axiom of expected utility theory described earlier. Such phenomena suggest that expected utility theory does not fully reflect decision-making in reality (Slovic and Tversky 1974).

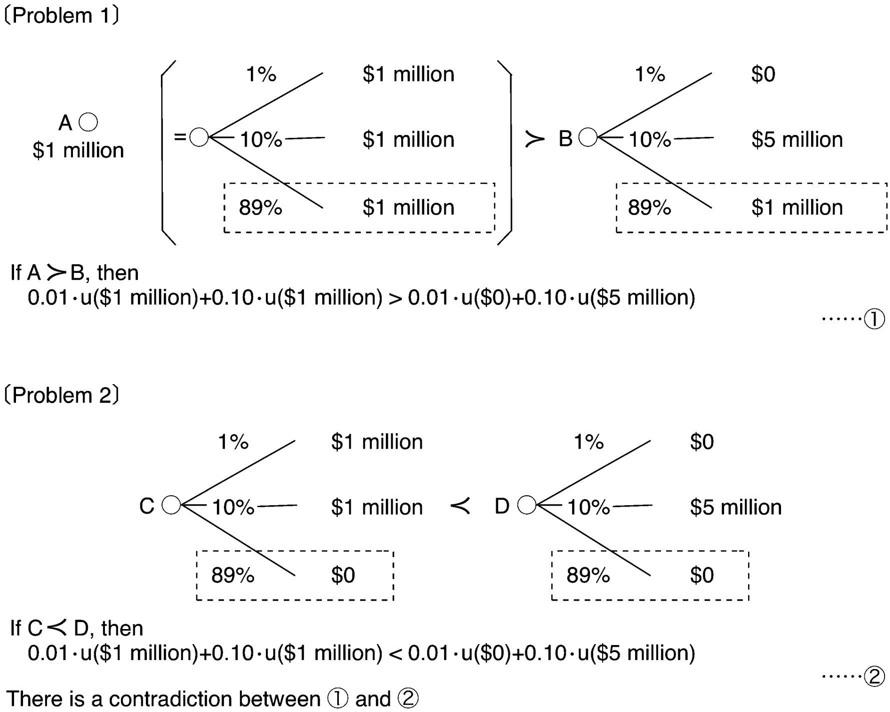

Fig. 6.1 Allais paradox. *Source*: Takemura (1996)

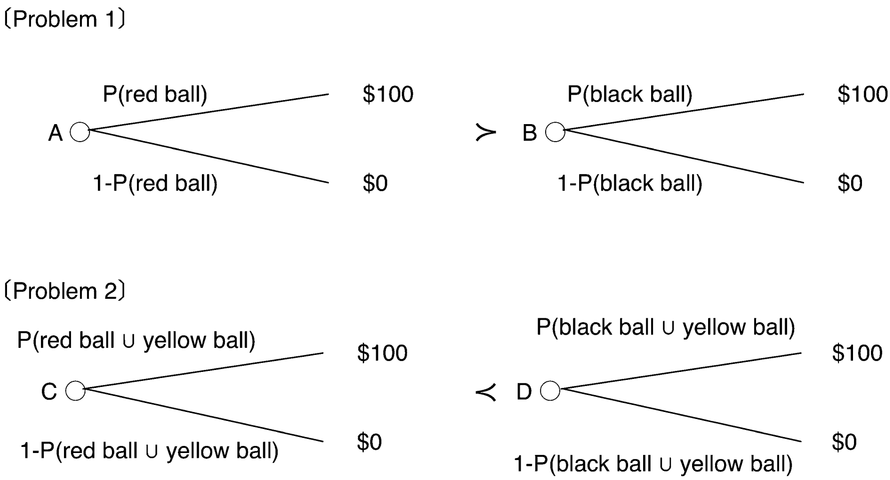

Fig. 6.2 Ellsberg's paradox. *Source*: Takemura (1996)

3.1 The Allais Paradox

Allais (1953) presented counterexamples of expected utility theory (Takemura 1996). We assume the following decision-making problem. First, Problem 1 is related to the selection between Alternatives A and B, as shown in Fig. 6.1. Selecting Alternative A results in a definite gain of $1 million. Alternative B is a lottery that offers a gain of $5 million with a probability of 10 %, $1 million with a probability of 89 % and $0 (no prize money) with a probability of 1 %. A comparison of A and B is likely to lead most people to select A, which promises a definite gain of prize money.

Subsequently, Problem 2 is one of two lotteries, which are Alternative C that provides $1 million with a probability of 11 % and Alternative D that offers a gain of $5 million with a probability of 10 %. In this case, many people presumably prefer D to C.

This result, however, clearly contradicts expected utility theory. The reason is that, first, the sections enclosed by dashed rectangles are common among problems. Therefore, they need not to be considered in the preference based on the independence axiom of expected utility theory and also in the sections without the dashed rectangle enclosures, A of Problem 1 and C of Problem 2 are the same and B of Problem 1 and D of Problem 2 are the same (see Fig. 6.1).

The Allais paradox has been indicated by numerous test subjects in psychological experiments (Slovic and Tversky 1974; Tversky and Kahneman 1992), which is thought in psychology to be a result of the certainty effect that a definite gain is preferred to an uncertain gain.

3.2 The Ellsberg Paradox

Ellsberg (1961) expressed preference in connection with ambiguity in the case in which the probability distribution of the results is unknown using specific examples and presented counterexamples of expected utility theory (subjective expected utility theory) (Takemura 1996).

We consider the following situation by following the paradox presented by Ellsberg (see Fig. 6.2). We know that a total of 90 balls are in a pot, of which 30 are red balls and 60 are a mix of black and yellow balls, the composition ratio of which is unknown. Assuming that one ball is taken out of this pot, we consider the following decision-making problem.

As presented in Fig. 6.2, in Problem 1, Alternative A is a gamble that yields a gain of $100 if a red ball ($r$) comes out and $0 if any other color comes out. The other Alternative B is a gamble that provides a gain of $100 if a black ball ($b$) comes out and $0 for any other color. A comparison of these two alternatives is likely to lead most people to prefer A to B ($A \succ B$).

Next, as shown in Fig. 6.2, in Problem 2, Alternative C provides \$100 if a red or yellow ball (r or y) comes out and \$0 for any other color. Alternative D is a gamble yielding a gain of \$100 if a black or yellow ball (b or y) comes out and \$0 for any other color. In this case, most people are likely to prefer D to C ($D \succ C$).

Such results of preference, however, clearly contradict expected utility theory, which assumes additivity of probability that a sum event of mutually exclusive events equals the sum of probabilities of individual events. In other words, the preference ($A \succ B$) in Problem 1 means that the probability $P(r)$ of taking out a red ball is higher than the probability $P(b)$ of picking a black ball ($P(r) \succ P(b)$); the preference ($D \succ C$) in Problem 2 means that the probability $P(r \cup y)$ of picking a red or yellow ball is lower than the probability $P(b \cup y)$ of taking out a black or yellow ball ($P(r \cup y) < P(b \cup y)$). The pair of r and y and the pair of b and y are mutually exclusive events. Therefore, $P(r \cup y) = P(r) + P(y)$ and $P(b \cup y) = P(b) + P(y)$ when additivity of probability is assumed.

Based on this, the preference, $D \succ C$ in Problem 2 indicates $P(b) > P(r)$, clearly contradicting the conclusion $P(r) > P(b)$ from the preference in Problem 1.

This Ellsberg paradox can be interpreted as implying a deviation from the independence axiom in expected utility theory. One conceivable psychological cause of this Ellsberg paradox is ambiguity aversion, an attitude of a decision maker to seek to avoid ambiguity. In other words, this is an attitude of people to dislike ambiguity when the probability of the results is unknown and avoid selecting the ambiguous alternative. In recent years, various explanations have been proposed for why such aversion arises. Numerous empirical studies have been conducted.

References

Allais, M. (1953). Le comportement de l'homme rationnel devant le risque: Critique des postulates et axiomes de l' ecole Americaine. *Econometrica, 21*, 503–546.

Barberà, S., Hammond, P. J., & Seidl, C. (Eds.). (1998). *Handbook of utility theory, vol. 1: Principles*. Dordrecht: Kluwer Academic Publishers.

Bell, D. E., Raiffa, H., & Tversky, A. (1988). Descriptive, normative, and prescriptive interactions in decision making. In D. E. Bell, H. Raiffa, & A. Tversky (Eds.), *Decision making: Descriptive, normative, and prescriptive interactions* (pp. 9–30). New York: Cambridge University Press.

Edwards, W. (Ed.). (1992). *Utility theories: Measurement and applications*. Boston: Kluwer Academic Publishers.

Ellsberg, D. (1961). Risk, ambiguity, and the Savage axiom. *Quarterly Journal of Economics, 75*, 643–669.

Fishburn, P. C. (1988). *Nonlinear preference and utility theory*. Sussex: Wheatsheaf Books.

Ichikawa, A. (1983). *Ishiketteiron: Enjiniaringu saiensu kouza33 [Decision-making theory: Engineering science course 33]*. Tokyo: Kyoritsu Publishing.

Iverson, G., & Luce, R. D. (1998). The representational measurement approach to problems. In M. H. Birnbaum (Ed.), *Measurement, judgment, and decision making* (pp. 1–79). San Diego: Academic.

Jensen, N. E. (1967). An introduction to Bernoullian utility theory. I. Utility functions. *Swedish Journal of Economics, 69*, 163–183.

Savage, I. R. (1954). *The foundations of statistics*. New York: Wiley.

Schultz, D. (1981). *A history of modern psychology* (3rd ed.). New York: Academic [Murata, K. (Trans.). (1986). Gendai shinrigaku no rekishi. Tokyo: Baifukan].

Slovic, P., & Tversky, A. (1974). Who accepts savage's axiom? *Behavioral Science, 19*, 368–373.

Takemura, K. (1996). Ishikettei to sono shien [Decision-making and support for decision-making]. In S. Ichikawa (Ed.), *Ninchi shinrigaku 4kan shikou [Cognitive psychology vol. 4 thoughts]* (pp. 81–105). Tokyo: University of Tokyo Press.

Tamura, H., Nakamura, Y., & Fujita, S. (1997). *Koyo bunseki no suri to oyo [Mathematical principles and application of utility analysis]*. Tokyo: Corona Publishing.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5*, 297–323.

von Neumann, J., & Morgenstern, O. (1944/1947). *Theory and games and economic behavior*. Princeton: Princeton University Press.

Part IV
Decision Making and Prospect Theory

Chapter 7
Preference Paradox and Nonlinear Expected Utility Theory

The previous chapter explained that expected utility theory included counterexamples called the Allais paradox (Allais 1953) and the Ellsberg paradox (Ellsberg 1961). The Allais and Ellsberg paradoxes are interpreted as deviations from the independence axiom. This chapter first explains the relations between these paradoxes and the independence axiom.

In recent years, these paradoxes have become explainable using such theory systems called nonlinear utility theory (Fishburn 1988; Edwards 1992), which does not assume this independence axiom, and generalized expected utility theory (Quiggin 1993). Prospect theory as proposed by Kahneman and Tversky (1979), Tversky and Kahneman (1992) is a theory that particularly integrates knowledge and past findings in behavioral decision-making theory and nonlinear utility theory (or generalized expected utility theory). This chapter explains the idea of non-additive probability assumed in nonlinear utility theory and the expected utility based on such non-additive probability. The chapter finally describes the basic assumptions in prospect theory.

1 Relations Between Independence Axiom and Paradoxes

The Allais and Ellsberg paradoxes can be described using deviations from the independence axiom in expected utility theory. Because the Allais paradox is a paradox of decision-making under risk, the probability distribution in the state of nature is known. The Ellsberg paradox, on the other hand, is the case in which, generally, only the state of nature is known, constituting a problem under uncertainty.

In the case of decision-making under risk, the independence axiom demands that if $p \succ t$ for arbitrary probability distributions p, t, and r, then the preference relation between the convex combination $(\lambda p + (1 - \lambda)r)$ of probability distributions, p and r, and the convex combination, $\lambda t + (1 - \lambda)r$, of t and r are the same. In other words, for all probability distributions $p, t, r \in P_x$ and for all probabilities $0 < \lambda < 1$,

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_7,

$$p \succ t \Rightarrow \lambda p + (1-\lambda)r \succ \lambda t + (1-\lambda)r$$

holds true. Therefore, if the independence axiom cannot be established, then a certain probability distribution $p, t, r \in P_x$ and a probability $0 < \alpha < 1$ exist. Furthermore, $\alpha t + (1-\alpha)r \succsim \alpha p + (1-\alpha)r$ holds in spite of $p \succ t$ (Tamura et al. 1997).

1.1 Independence Axiom Under Risk

Daniel Ellsberg

Daniel Ellsberg was born in Detroit, Michigan, the United States of America in 1931. After graduating from Harvard University Department of Economics in 1952, he attended the University of Cambridge, served as a U.S. Marine between 1954 and 1957, and worked as a Harvard University Junior Fellow between 1957 and 1959. Subsequently in 1962, he earned a doctoral degree from the same university. In his doctoral dissertation, he introduced the so-called Ellsberg paradox and pointed out the problems of expected utility theory. His studies can be considered the pioneer of the current nonlinear utility theory and behavioral decision-making theory. Many of his numerous papers are still studied today. In 2002, he was invited into the Society for Judgment and Decision Making for behavioral decision-making theory, at which time he presented a lecture entitled “the Allais and Ellsberg Paradoxes: 40 Years Later.” The author attended this lecture in this occasion, but he described decision-making research only a little and spoke for an extended time to deliver his anti-war message.

In fact, Ellsberg is well-known also as a historic anti-war activist who made an accusation against the problems of the Vietnam War. The following gives a brief background in this regard. Ellsberg worked at a military-related job at a research institute, The RAND Corporation, in 1959, and joined the U.S. Department of Defense in 1964. After serving as a special assistant to the Assistant Secretary of Defense for International Security Affairs of the Department of Defense, he was sent to Vietnam in 1965. In 1967, he returned to The RAND Corporation as a fellow researcher while working under Robert McNamara, then Secretary of Defense. While working at these jobs, he gradually became skeptical of the direction of the Vietnam War. In 1969, he began making copies of secret documents, eventually collecting 7,000 pages that vividly reflected the history of the U.S. government’s deception of the people. In 1971, he handed the copied documents to the media such as the Washington Post and New York Times for informing people of the truth.

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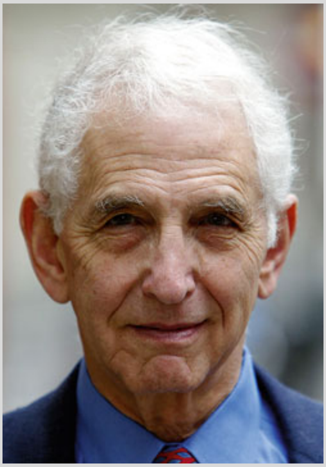
These documents are known as the Pentagon Papers, the contents of which were published in newspapers. Ellsberg was arrested for this action and charges were brought against him (the prosecution was dismissed in 1973). He was arrested more recently for sitting in and refusing to leave the park in front of the White House in his anti-Iraq War action in 2003. His website is http://www.ellsberg.net/.



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In Problem 1 of the Allais paradox, selecting Alternative A provides a definite gain of \$1 million and Alternative B is a selection of a lottery that results in a gain of \$5 million with a probability of 10 %, \$1 million with a probability of 89 %, or \$0 (no prize money) with a probability of 1 % (see Chap. 6). Alternative A can be broken down into \$1 million with a probability of 10 %, \$1 million with a probability of 89 %, and \$1 million with a probability of 1 %. Consequently, the common factor between A and B is that \$1 million can be gained with a probability of at least 89 %. If Alternative A is expressed as p and Alternative B as q, and a lottery that offers \$5 million with a probability of ten-elevenths (10/11) while providing no gain at all with a probability of one-eleventh (1/11) is expressed as t, the relations can be expressed as

$$p = 0.11p + 0.89p \text{ and}$$
$$q = 0.11t + 0.89p.$$

Therefore, according to the independence axiom, if $p \succ t$, then $p \succ q$ will result.

Problem 2 involves two lotteries—Alternative C, which offers \$1 million with a probability of 11 %, and Alternative D, which offers \$5 million with a probability of 10 %. They share a common factor that there is at least 89 % probability of gaining nothing at all. If Alternative C is expressed as r, Alternative D as s, and a lottery that definitely provides \$0 as t', then the relations are expressed as shown below.

$$r = 0.11p + 0.89t'$$

$$s = 0.11t + 0.89t'$$

Therefore, if $p \succ t$, then $r \succ s$, based on the independence axiom.

The discussion up to this point can be summarized that the independence axiom demands that if $p \succ t$, then $p \succ q$ and $r \succ s$, and if $t \succ p$, then $p \succ q$ and $s \succ r$. In the actual selection however, the test subjects express $p \succ q$ and $s \succ r$ (Slovic and Tversky 1974), which therefore does not satisfy the independence axiom.

1.2 Independence Axiom Under Uncertainty

The independence axiom under uncertainty is the following (Tamura et al. 1997): first, we assume that X is the set of results, Θ is the set of states of nature, $A \subseteq \Theta$ represents events, and two alternatives are $f : \Theta \to X$ and $g : \Theta \to X$. The independence axiom demands that if $f(\theta) = g(\theta)$ for arbitrary $\theta \notin A$, then the preference relation between f and g is independent of a complementary event A^c. Therefore, if the independence cannot be established, the following is true. If certain alternatives f, g, f', and g' are $\theta \in A$ for a given event A, then $f(\theta) = f'(\theta)$ and $g(\theta) = g'(\theta)$; furthermore, if $\theta \notin A$ when $f(\theta) = g(\theta)$ and $f'(\theta) = g'(\theta)$, then $g' \succsim f'$ in spite of $f \succ g$.

Let us explain this point using the Ellsberg paradox. We know that a total of 90 balls are placed in a pot, of which 30 are red and 60 are a mix of black and yellow balls. The composition ratio of the black and yellow balls is unknown. In this problem of decision-making under uncertainty, the decision-maker is assumed to confront some type of probability p. In Problem 1, Alternative A is a gamble that results in a gain of \$100 if a red ball (r) comes out and \$0 if any other color such as black (b) or yellow (y) comes out. Alternative B is a gamble that yields a gain of \$100 if a black ball (b) comes out and \$0 when any other color emerges. When Alternative A is expressed as f and Alternative B is expressed as g:

$$\text{Expected utility of } f = p(r)u(100) + p(b \cup y)u(0)$$
$$\text{Expected utility of } g = p(b)u(100) + p(r \cup y)u(0).$$

Because the Ellsberg paradox assumes $f \succ g$,

$$f \succ g \Leftrightarrow \text{ expected utility of } f > \text{expected utility of } g$$

$$p(r)u(100) + p(b \cup y)u(0) > p(b)u(100) + p(r \cup y)u(0).$$

Because p is a probability, additivity holds for mutually exclusive events. Moreover, because $u(100) > u(0)$ can be assumed, the following can be established.

$$\begin{aligned}
& f \succ g \\
& \Leftrightarrow p(r)u(100) + p(b)u(0) + p(y)u(0) > p(b)u(100) + p(r)u(0) + p(y)u(0) \\
& \Leftrightarrow p(r)u(100) + p(b)u(0) - p(b)u(100) - p(r)u(0) > 0 \\
& \Leftrightarrow (p(r) - p(b))(u(100) - u(0)) > 0 \\
& \Leftrightarrow p(r) > p(b).
\end{aligned}$$

Similarly, Problem 2 assumes Alternative C constituting a gamble that yields a gain of \$100 if a red or yellow ball (r or y) comes out and \$0 if any other color comes out. The other Alternative D is a gamble that provides \$100 for a black or yellow ball (b or y) that comes out and \$0 for any other color. Because the preference is $g' \succ f'$ when Alternative C is expressed as f' and Alternative D as g', then

$$g' \succ f' \Leftrightarrow p(b) > p(r) \text{ must hold true.}$$

This result clearly contradicts $p(r) > p(b)$ and indicates that $g \succ f$ and $g' \succ f'$ cannot be established simultaneously. The result also suggests that expected utility theory cannot explain the Ellsberg paradox irrespective of how the subjective probability is set.

The fact that the Ellsberg paradox does not satisfy the independence axiom in decision-making under uncertainty is evident in Table 7.1. In other words, the invalidity of independence means that, given that alternatives f, g, f', and g' are $\theta \in A$ for a given event A (red, or black), then $f(\theta) = f'(\theta)$ and $g(\theta) = g'(\theta)$. Furthermore, if $\theta \notin A$ (if θ is yellow), then $g' \succ f'$ when $f(\theta) = g(\theta)$ and $f'(\theta) = g'(\theta)$ in spite of $f \succ g$. Therefore, the Ellsberg paradox indicates $g' \succ f'$ in spite of $f \succ g$ and therefore does not satisfy the independence axiom.

Table 7.1 Ellsberg's paradox and the state of nature

	State of nature		
Alternative	Red (r) (in\$)	Black (b) (in\$)	Yellow (y) (in\$)
f	100	0	0
g	0	100	0
f′	100	0	100
g′	0	100	100

2 Non-additive Probability and Nonlinear Utility Theory

Both the Allais and Ellsberg paradoxes can be understood as being caused by the empirical invalidity of the independence axiom. Various theoretical frameworks explain such paradoxes (Camerer et al. 2004; Einhorn and Hogarth 1986; Nakamura 1992; Takemura 2000; Seo 1994; Tamura et al. 1997; Takemura 1996a).

A salient example is the explanation based on nonlinear utility theory that has relaxed the independence axiom and other factors. This body of theory forms the generalization of expected utility theory (Starmer 2000; Tamura et al. 1997). Although this body of theory is called nonlinear utility theory (Fishburn 1988; Edwards 1992) or generalized expected utility theory (Quiggin 1993) in the field of economics, it is nearly equivalent to the body of theory related to fuzzy integrals in fuzzy measure theory in the field of engineering (Sugeno and Murofushi 1993).

The body of nonlinear utility theory often assumes a non-additive probability weighting function that converts probabilities for which additivity does not hold even if probability information is given for decision-making under risk, such as in the case of the Allais paradox. In the case of the Ellsberg paradox, non-additive probabilities in which additivity does not hold for the measure of subjective belief in a state of nature are formulated.

A non-additive probability is sometimes expressed as a "capacity," but is called, in some cases, "fuzzy measure" in the field of engineering. Its mathematical definition is the same despite the varying names. A non-additive probability refers to a set function, $\pi : 2^{\Omega} \rightarrow [0,1]$ from an aggregate consisting of a subsets of a nonempty set, Ω, to a closed interval, [0,1], which is also a set function that satisfies both a boundedness condition ($\pi(\phi) = 0$, $\pi(\Omega) = 1$) and a monotonicity condition (if the relation of subsets E and F of Ω is $E \subseteq F$, then relation $\pi(E) \leq \pi(F)$ is satisfied). A non-additive probability is so named because it does not necessarily satisfy the conditions of additivity.

In the problems of Ellsberg, too, if boundedness conditions $\pi(\phi) = 0$ and $\pi(r \cup b \cup y) = 1$ are assumed and additionally a monotonicity condition is assumed for the assessment of probability, then the paradox does not necessarily occur. Whereas relations such as $\pi(r \cup b \cup y) > \pi(b \cup y) > \pi(r) > \pi(\phi)$ must be satisfied based on the monotonicity condition, even assumptions such as $\pi(r) = 1/3$, $\pi(b \cup y) = 2/3$, $\pi(b) < 1/3$, and $\pi(r \cup y) < 2/3$ would not deviate from the conditions of non-additive probability. Therefore, no contradiction occurs in problems 1 and 2 of Ellsberg. In this case, however, this non-additive probability is $\pi(r) + \pi(b) < \pi(y \cup b)$, $\pi(y) + \pi(r \cup b) < \pi(y \cup r \cup b)$, which satisfies the conditions of superadditivity.

In expected utility theory, although the criterion of expected utility maximization can be viewed from the perspective of Lebesgue integration related to a probability measure, there are a few ways of integral expression other than Lebesgue integration for the expected utility related to a non-additive probability as defined above. In the field of fuzzy measure theory in engineering, integrals are expressed in a few ways in view of fuzzy integrals (Sugeno and Murofushi 1993). Of these expressions, those that are studied enthusiastically by researchers of nonlinear utility theory and fuzzy theory are the expected utility using Choquet integrals (Choquet 1954). The expected utility theory based on this integral is also called rank-dependent utility theory.

The expected utility using Choquet integrals can be expressed as follows (Quiggin 1993; Camerer 1995): first, we assume that a state of nature $\theta_i \in \Omega$ is ranked as $u(f(\theta_1)) > u(f(\theta_2)) > \cdots > u(f(\theta_n))$ according to the utility $u(f(\theta_i))$ of the result $f(\theta_i)$ of an alternative, f. The expected utility using Choquet integrals on a finite set related to non-additive probability π is

$$u(f(\theta_1))\pi(\theta_1) + \sum_{i=2}^{n} u(f(\theta_i))\left[\pi\left(\bigcup_{j=1}^{i}\theta_j\right) - \pi\left(\bigcup_{j=1}^{i-1}\theta_j\right)\right].$$

If π is an additive measure and a state of nature θ_i is mutually exclusive, then the expected utility shown above becomes consistent with that in subjective expected utility theory (Camerer 1995).

Are non-additive probabilities appropriate as a measure for assessment of people's subjective uncertainty? To begin with the conclusion, even a non-additive probability assuming only monotonicity without the condition of additivity might be too difficult in its psychological aspects. The condition of monotonicity is not as mathematically difficult as a probability measure that assumes additivity and might be regarded as applicable, in general, to people's judgment. Psychologically, however, cases in which the condition of monotonicity is invalid are occasionally observed.

Tversky and Kahneman (1983) found that when given a description of an intelligent and active 31-year-old single woman called Linda, more test subjects estimated that the probability of an event that "she is currently a bank teller and is enthusiastic about women's rights movement" would be higher than the probability of the event that "she is currently a bank teller." This result means that the subjective probability, $\pi(t \cap w)$, of the product set of the event, (t), that Linda is a bank teller and the event, (w), that she is enthusiastic about the women's rights movement is assessed higher than the subjective probability ($\pi(t)$) that she is a bank teller. However, assuming that a subjective probability satisfies the condition of monotonicity, then the subjective probability of a product event must be below the subjective probability of a single event (because $\pi(t \cap w) \leq \min(\pi(t), \pi(w))$ based on the condition of monotonicity). Therefore, the result implies that the assessment of many of the test subjects in their study deviated from the condition of monotonicity.

Such assessment is named a conjunction fallacy as a bias in the judgment of a conjunctive event and has been eagerly studied by psychologists. This conjunction fallacy has been found to occur easily even in the judgment that allows the ambiguity of fuzzy rating for expressing a probability with an interval such as an upper limit and lower limit (Takemura 1996b). Probability judgment that does not satisfy such monotonicity means that the expression with a non-additive probability assuming monotonicity has a limitation as a descriptive theory. Murofushi et al. (1994) proposes a Choquet integral of a measure that has a non-monotonous nature, and empirical research on the judgment and decision-making based on this theory is expected to develop in the future.

Although the expression of expected utility using Choquet integrals is also adopted in the body of fuzzy theory and expected utility theory, it is also used in the body of psychological descriptive theory of decision-making called prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992).

3 Basic Assumptions of Prospect Theory

Prospect theory was proposed by Kahneman and Tversky, which combines the knowledge of behavioral decision-making theory and nonlinear utility theory (or, generalized expected utility theory). Prospect theory was proposed initially as a descriptive theory that handles decision-making under risk (Kahneman and Tversky 1979). It was subsequently developed into a theory that can explain decision-making under uncertainty (Tversky and Kahneman 1992).

The word *prospect* in prospect theory refers to the various results of selecting an alternative and combinations of probabilities that correspond to the results, which is equivalent to the "gamble" in decision-making under risk. In decision-making under risk, a desirable prospect is selected from several prospects. In other words, assuming the set $X = \{x_1, \ldots, x_j, \ldots, x_m\}$ of results that occur, this can be replaced with a problem of which of the probability distributions, $p_1 = [p_{11}, p_{12}, \ldots, p_{1m}]$, $p_2 = [p_{21}, p_{22}, \ldots, p_{2m}], \ldots, p_l = [p_{l1}, p_{l2}, \ldots, p_{lm}]$ on X is selected. At this point, one prospect is expressed as $(x_1, p_{l1}; \ldots; x_j, p_{lj}; \ldots; x_m, p_{lm})$. Prospect theory assumes that this prospect is assessed in a way that differs from expected utility theory.

In prospect theory, the decision-making process is divided into the editing phase, in which a problem is recognized and the framework for decision-making is determined, and the evaluation phase, in which alternatives are evaluated according to the recognition of the problem (Kahneman and Tversky 1979). The former phase is situation-dependent and varies depending even on a slight difference in linguistic expressions. In the latter phase, on the other hand, evaluation and decision-making are independent of the situation once a problem is identified.

3.1 Editing Phase

The editing phase is a stage at which alternatives are cognitively restructured, and the recognition even of the same decision-making problem varies depending on the framing of the problem, which can change because of even a slight difference in linguistic expressions. In the editing phase, mental operations including (1) coding, (2) combination, (3) segregation, (4) cancellation, (5) simplification, and (6) detection of dominance are conducted.

1. Coding is a mental operation that divides results into either a gain or loss. This is a case, for example, in which a part-time worker usually making \$8 per hour would perceive a sudden raise to \$9 per hour as a gain and a cut to \$7 per hour as a loss. In this case, the normal wage of \$8 per hour functions as a reference point.
2. Combination is a mental operation in which the same gains are combined and simplified. For example, the prospect (200, 0.25; 200, 0.25) in which a probability of gaining \$200 is 0.25 and another probability of gaining \$200 is 0.25 can be edited as the prospect (200, 0.50) whose probability of gaining \$200 is 0.50.

3. Segregation is a mental operation that separates the portion of definite gain and the portion of risky gain. For example, the prospect (300, 0.80; 200, 0.20) in which a probability of gaining \$300 is 0.80 and a probability of gaining \$200 is 0.20 presents a separation between the prospect (200, 1.00) with a definite gain of \$200 and the prospect (100, 0.80) entailing a probability of gaining \$100 (0.80).
4. Cancellation is a mental operation in which two prospects being compared are perceived with common elements being ignored. For example, the prospect (200, 0.20; 100, 0.50; − 50, 0.30) and the prospect (200, 0.20; 150, 0.50; − 100, 0.30) are perceived by being reduced to the prospect (100, 0.50; − 50, 0.30) and the prospect (150, 0.50; − 100, 0.30).
5. Simplification is a mental operation that rounds and simplifies results and their probabilities. For example, the prospects (101,0.49) are simplified to (100,0.50) to be understood.
6. Detection of dominance is a mental operation in which dominant alternatives are detected. If, for example, the send elements of both the prospect (500, 0.20; 101, 0.49) and the prospect (500, 0.15; 99, 0.51) are simplified to (100, 0.50), the comparison would be between the prospects (500, 0.20) and (500, 0.15); the former is found to be dominant over the latter.

3.2 Evaluation Phase

Each prospect is restructured in the editing phase, based on which the prospect that has been evaluated the highest is selected in the evaluation phase. In the evaluation phase, the prospects are evaluated based on a type of utility function that they call the value function and the probability weighting function. This method of evaluating in the evaluation phase is basically the same as the rank-dependent utility theory in nonlinear utility theory.

As depicted in Fig. 7.1, the value function is a concave function in the area of gain and thus risk-averse and is a convex function in the area of loss and risk-taking. In addition, the slope of the value function is generally steeper in the area if loss than in the area of gain, which implies that a loss would have a greater impact than a gain.

A special aspect of prospect theory is the assumption that the part corresponding to the origin of utility theory is the reference point, which easily shifts depending on how the decision-making problem is edited in the editing phase. In prospect theory, the results are evaluated based on their deviation from the reference point representing the psychological origin; the decision-maker evaluates the results either as a gain or loss. Furthermore, prospect theory assumes the decision-maker’s evaluation of a gain as risk aversion and a loss as risk-taking. Even the same decision-making problem shows risk-aversion when the alternatives are perceived in the area of gain; it also shows risk taking when in the area of loss.

Moreover, a non-additive probability weighting function in prospect theory is $\pi(0) = 0$, $\pi(1) = 1$, whose form is presented in Fig. 7.2. Assuming that this

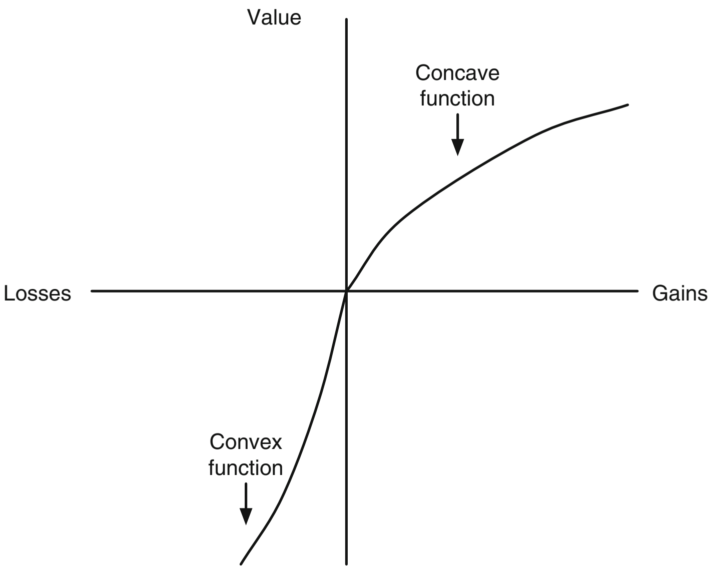


Fig. 7.1 Value function in prospect theory. *Source*: Kahneman and Tversky (1979). Reproduced in part by author

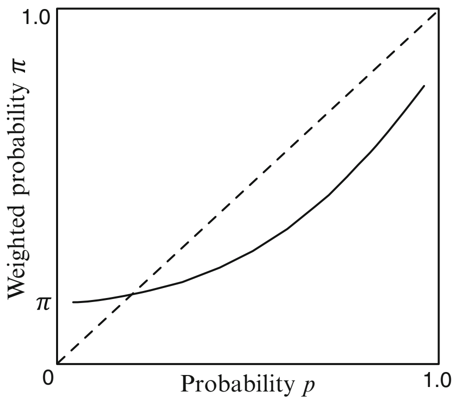


Fig. 7.2 Probability weighting function in prospect theory. *Source*: Kahneman and Tversky (1979). Reproduced in part by author

probability weighting function is π and the objective probability is p, this probability weighting function has the following qualitative characteristics.

(i) It satisfies the condition of $\pi(p) + \pi(1 - p) \leq 1$.
(ii) It overvalues the probability when the probability is very low, engendering the relation of $\pi(p) > p$.
(iii) It shows non-proportionality, i.e. $\dfrac{\pi(pq)}{\pi(p)} \leq \dfrac{\pi(pqr)}{\pi(pr)}$.
(iv) It indicates non-continuity near the endpoints.

Finally, the method of prospect evaluation in prospect theory can be described as follows: According to Kahneman and Tversky (1979), assuming that x and y are the results, p and q are the probabilities of each respective result, $\pi(p)$ and $\pi(q)$ are the probability weights for p and q, and $v(x)$ and $v(y)$ are the values of each result, the value of evaluated prospect $V(x,p;y,q)$ is the following:

If any one of $p + q < 1$, $x \geq 0 \geq y$ or $x \leq 0 \leq y$ holds and $v(0) = 0$, then
$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y)$.
If $p + q = 1$ and $x > y > 0$ or $x < y < 0$, then
$V(x, p; y, q) = v(y) + \pi(p)(v(x) - v(y))$ holds.

Forecasting based on prospect and theory and research on this subject will be described in the next chapter.

References

Allais, M. (1953). Le comportement de l'homme rationnel devant le risque; critique des postulats et axiomes de l'ecole americaine. *Econometrica, 21*, 503–546.

Camerer, C. (1995). Individual decision making. In J. H. Hagel & A. E. Roth (Eds.), *Handbook of experimental economics* (pp. 587–703). Princeton: Princeton University Press.

Camerer, C. F., Lowenstein, G., & Rabin, M. (Eds.). (2004). *Advances in behavioral economics*. Princeton: Princeton University Press.

Choquet, G. (1954). Theory of capacities. *Annales de l' Institute Fourier, 5*, 131–295.

Edwards, W. (Ed.). (1992). *Utility theories: Measurements and applications*. Boston: Kluwer Academic Publishers.

Einhorn, H., & Hogarth, R. (1986). Decision-making under ambiguity. *Journal of Business, 59*, 225–250.

Ellsberg, D. (1961). Risk, ambiguity, and the savage axiom. *Quarterly Journal of Economics, 75*, 643–669.

Fishburn, P. C. (1988). *Nonlinear preference and utility theory*. Sussex: Wheatsheaf Books.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47*, 263–292.

Murofushi, T., Sugeno, M., & Machida, M. (1994). Nonmonotonic fuzzy measures and choquet integral. *Fuzzy Sets and System, 64*, 73–86.

Nakamura, K. (1992). On the nature of intransitivity in human preferential judgments. In V. Novak, J. Ramik, M. Mares, M. Cherny, & J. Nekola (Eds.), *Fuzzy approach to reasoning and decision making* (pp. 147–162). Dordrecht: Kluwer.

Quiggin, J. (1993). *Generalized expected utility theory: The rank dependent model*. Boston: Kluwer Academic Publishers.

Seo, F. (1994). *Shiko no Gijutu: Aimai kankyoka no keiei ishikettei [Thinking techniques: Management decision making under ambiguity environment]*. Tokyo: Yuhikaku Publishing.

Slovic, P., & Tversky, A. (1974). Who accepts savage's axiom? *Behavioral Science, 19*, 368–373.

Starmer, C. (2000). Developments in non-expected utility theory: The hunt for descriptive theory of choice under risk. *Journal of Economic Literature, 38*, 332–382.

Sugeno, M., & Murofushi, T. (1993). *Koza faji 3: Faji sokudo [Course fuzzy 3: Fuzzy measure]*. Tokyo: The Nikkan Kogyo Shimbun.

Takemura, K. (1996a). Ishikettei to sono shien [Decision-making and support for decision-making]. In S. Ichikawa (Ed.), *Ninchi shinrigaku 4kan shikou [Cognitive psychology, vol. 4: Thoughts]* (pp. 81–105). Tokyo: University of Tokyo Press.

Takemura, K. (1996b). *Ishikettei no shinri: Sono katei no tankyu [Mentality of decision-making: The quest for the process]*. Tokyo: Fukumura Shuppan Inc.

Takemura, K. (2000). Vagueness in human judgment and decision making. In Z. Q. Liu & S. Miyamoto (Eds.), *Soft computing for human centered machines* (pp. 249–281). Tokyo: Springer.

Tamura, H., Nakamura, Y., & Fujita, S. (1997). *Koyo bunseki no suri to oyo [Mathematical principles and application of utility analysis]*. Tokyo: Corona Publishing.

Tversky, A., & Kahneman, D. (1983). Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment. *Psychological Review, 90*, 293–315.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5*, 297–323.

Chapter 8
Prospect Theory and Decision-Making Phenomena

Prospect theory, proposed by Kahneman and Tversky (1979) (Tversky and Kahneman 1992), is a theory that combines the knowledge of behavioral decision theory and nonlinear utility theory (or generalized expected utility theory). This chapter first reports the forecasts derived from the basic assumptions of prospect theory and empirical examples of such forecasts. It subsequently provides an overview of a new version of cumulative prospect theory that explains decision-making not only under risk but also under uncertainty and in light of experimental research into this theory.

1 Empirical Research on the Value Function and Reflection Effect

Prospect theory assumes that the value function $v(x)$ is a concave function (a function that is concave downward) in the area of gain above the reference point and is a convex function (a function that is convex downward) in the area of loss below the reference point, which is $v''(x) < 0$ when $x > 0$ and $v''(x) > 0$ when $x < 0$. This relation reflects the decision-maker's risk-averse behavior in the area of gain and risk-taking behavior in the area of a loss.

Kahneman and Tversky (1979) distributed questionnaires to students and instructors at universities in Israel, the U.S., and Sweden and examined this assumption related to the value function.

Problem 1 Which one of the following alternatives is preferred to the other?

A. a gain of $4,000 with a probability of 80 % (*Prospect A* = (4000, 0.80))
B. a certain gain of $3,000 (*Prospect B* = (3000, 1.00))

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_8,

Problem 2 Which one of the following alternatives is preferred to the other?

C. a loss of $4,000 with a probability of 80 % (*Prospect C* = (−4000, 0.80))
D. a certain loss of $3,000 (*Prospect D* = (−3000, 1.00))

For Problem 1, 20 % of 95 respondents selected A; 80 % preferred B. Regarding Problem 2, 92 % of 95 respondents preferred C; 8 % opted for D. This pattern of the majority selection is consistent with the forecasts of prospect theory that are risk-averse in the area of gain and risk-taking in the area of loss. Such a phenomenon in which the risk attitude is reversed between the areas of gain and loss is called the reflection effect.

Kahneman and Tversky also reported, based on the response patterns below, the phenomenon in which preference is reversed by the reflection effect even for prospects that have identical final results.

Problem 3 You first receive $1,000. Select either of the alternatives below.

A. a gain of $1,000 with a probability of 50 % (*Prospect A* = (1000, 0.50))
B. a certain gain of $500 (*Prospect B* = (500, 1.00))

Problem 4 You first receive $2,000. Select either of the alternatives below.

C. a loss of $1,000 with a probability of 50 % (*Prospect C* = (−1000, 0.50))
D. a certain loss of $500 (*Prospect D* = (−500, 1.00))

Of 70 test subjects who responded to Problem 3, 16 % selected A; 84 % chose B. Of 68 test subjects who responded to Problem 4, 69 % preferred C; 31 % opted for D. This pattern of the majority selection is consistent with the forecasts of prospect theory that are risk-averse in the area of gain and risk-taking in the area of loss.

Focusing only on the final results, Problems 3 and 4 prove to be identical. In other words, the final results of A (=(2000, 0.50; 1000, 0.50)) and C and those of B (=(1500, 1.00)) and D are the same. This result suggests that the test subjects did not take the $1,000 and $2,000 initially received into consideration when making the judgment. Such an experimentally obtained result demonstrates that people make a judgment and decision based on the change from the reference point rather than the amount of the final assets and become risk-taking or risk-averse depending on the case.

Toshino (2004) explains the behavior of stock investment as an example of the value function of prospect theory. He used an example in which the price of stocks purchased for ¥1,000 (approximately $10) per share rises to ¥2,000 (approximately $20) and then drops to ¥1,500 (approximately $15), and despite the gain of ¥500 (approximately $5) per share from the purchase price, the stockholder keeps the stocks rather than selling it. This case is perceived as a loss of ¥500 (approximately $5) if the reference point is ¥2,000 (approximately $20), which tends to result eventually in a person's retaining the stocks without selling them. The act of retaining the stocks is more risk-taking than selling them to realize the gain or loss, which is, thereby, consistent with prospect theory. Furthermore, prospect theory states that the ¥500 (approximately $5) is perceived as a gain if the reference point is ¥1,000 (approximately $10), which often results in a risk-averse attitude and sale of the stocks.

A phenomenon that can be interpreted as indicating the reflection effect based on such a value function has been observed in the actual stock market. Odean (1998) analyzed stock trading data and reported that the period of keeping ownership of stocks was a median of 104 days when a gain was realized and 124 days when a loss was incurred. This result can be interpreted as showing that investors tend to become risk-averse and quickly sell their stocks when they are profitable. They become risk-taking and hold on to their stocks for a long period of time when the stocks represent a loss to the holder (Camerer 2000).

Such a pattern of investment behavior is known in the field of finance as the disposition effect (Shefrin and Statman 1985; Toshino 2004). This disposition effect is found not only in the field of finance, but in the housing market. In other words, by this phenomenon, the period for which homeowners retain their homes rather than selling them is longer while a loss is incurred because a decline in the housing prices has been reported, which is explainable with the disposition effect (Camerer 2000).

2 Empirical Research on the Value Function and Loss Aversion

The value function of prospect theory in the area of loss has a slope that is steeper than that in the area of gain, which, therefore, is $v'(x) < v'(-x)$ when $x > 0$, where v' is the derivative of the value function, v. That result implies that the impact of a loss is greater than that of a gain, which is called loss aversion.

From the property of loss aversion assumed in prospect theory, avoidance of a gamble with zero expected value is derived. For instance, in the selection of either a gamble for a gain of $10,000 with a probability of 50 % or a loss of $10,000 with a probability of 50 % (the expected value is $0) or not gambling at all (the expected value is $0), the option of not gambling results from the property of loss aversion.

When $x > y > 0$, the prospect $(y, 0.50; -y, 0.50)$ is preferred to the prospect $(x, 0.50; -x, 0.50)$ (Kahneman and Tversky 1979). In other words, both

$$v(y) + v(-y) > v(x) + v(-x)$$

and

$$v(-y) - v(-x) > v(x) - v(y)$$

hold true. Additionally when $y = 0$, then

$$v(x) < -v(-x)$$

can be derived.

A phenomenon related to this is the problem of the equity premium in the field of finance. Because the price fluctuations in the stock market are greater than those in the bond market, if the same return can be expected, in general, investment in bonds is likely to be preferred. This outcome is consistent with the property of loss aversion described earlier. Benartzi and Thaler (1995) authored a hypothesis based on the property of loss aversion of prospect theory that investments that can cause a large loss despite a potentially large profit such as the case of stocks are valued low, which explained the very high excess profitability (premium) in the stock market.

One of the phenomena that can be derived from the property of loss aversion is the endowment effect (Kahneman et al. 1990, 1991), which is a phenomenon by which the selling price of certain goods that are given and retained becomes higher than the purchase price in the case where the goods are not given. Put in simple terms, this is a phenomenon by which the goods that are initially retained become difficult to give away, which is interpreted in some cases as expressing the status quo bias.

Kahneman et al. conducted a series of experiments to verify the endowment effect. In one such experiment, they first randomly divided 77 students from Simon Fraser University into three groups: the "sell" condition group, "buy" condition group, and "select" condition group (Kahneman et al. 1990). The test subjects in the "sell" condition group were given coffee mugs and asked at what price they would sell them. The test subjects in the "buy" condition group were asked at what price they would buy those mugs, and those in the "select" condition group were provided with various prices and asked whether they would select the mugs or prefer to receive cash.

As a result, the median price of the "sell" condition group was \$7.12, that of the "buy" condition group was \$2.87, and that of the "select" condition group was \$3.12. Such price disparity is considered attributable to the reference point that was the state of possessing the mugs for the "sell" condition group and the state of not possessing the mugs for the "buy" and "select" condition groups.

3 Empirical Research on Probability Weighting Function

According to prospect theory, the probability is overvalued when the probability is very low, resulting in the relation, $\pi(p) > p$. Kahneman and Tversky (1979) asked the university students and instructors who were their test subjects the following questions and examined this assumption about the probability weighting function.

Problem 1 Which one of the following alternatives is preferred to the other?

A. a gain of \$5,000 with a probability of 0.1 % (*Prospect A* = (5000, 0.001))
B. a certain gain of \$5 (*Prospect B* = (5, 1.00))

Problem 2 Which one of the following alternatives is preferred to the other?

C. a loss of $5,000 with a probability of 0.1 % (*Prospect C* = (−5000, 0.001))
D. a certain loss of $5 (*Prospect D* = (−5, 1.00))

Of 72 test subjects, 72 % selected A and 28 % chose B for Problem 1. In addition, 17 % opted for C and 83 % preferred D for Problem 2.

This result reveals that, in Problem 1, the test subjects preferred the gamble for a profit with a very low probability more than the amount of expected value. In Problem 2, the test subjects preferred the amount of expected value to the gamble with a very low probability of a loss.

Based on the response pattern of most of the test subjects in Problem 1, the relation

$$\pi(0.001)v(5000) > v(5)$$

is presented. Assuming that the value function, v, of prospect theory is a concave function in the area of gain, the following relation holds.

$$\pi(0.001) > \frac{v(5)}{v(5000)} > 0.001$$

Similarly, based on the result of Problem 2, the relation

$$\pi(0.001)v(-5000) < v(-5)$$

is indicated. Assuming that the value function v of prospect theory is a convex function in the area of loss, the following relation holds.

$$\pi(0.001) > \frac{v(-5)}{v(-5000)} > 0.001$$

This result demonstrates that a probability is overvalued when it is very low and the relation, $\pi(p) > p$, is formed.

Subsequently, prospect theory assumes that non-proportionality, $\pi(pqr)/\pi(pr) > \pi(pq)/\pi(p)$ holds for a probability $0 \leq p, q, r \leq 1$. Kahneman and Tversky (1979) asked their test subjects the following questions and examined this property.

Problem 3 Which one of the following alternatives is preferred to the other?

A. a gain of $6,000 with a probability of 45 % (*Prospect A* = (6000, 0.45))
B. a gain of $3,000 with a probability of 90 % (*Prospect B* = (3000, 0.90))

Problem 4 Which one of the following alternatives is preferred to the other?

C. a gain of $6,000 with a probability of 0.1 % (*Prospect C* = (6000, 0.001))
D. a gain of $3,000 with a probability of 0.2 % (*Prospect D* = (3000, 0.002))

Of 66 test subjects who responded to Problem 3, 14 % selected A and 86 % chose B. Of 66 test subjects who responded to Problem 4, 73 % preferred C and 27 % opted for D. Assuming that the value function, v, is a concave function in the area of gain based on the results of Problems 3 and 4, the relation

$$\frac{\pi(0.001)}{\pi(0.002)} > \frac{v(3000)}{v(6000)} > \frac{\pi(0.45)}{\pi(0.90)}$$

can be derived. Assuming $p = 9/10$, $q = 1/2$, and $r = 1/450$ at this point, the relation

$$\frac{\pi(pqr)}{\pi(pr)} > \frac{\pi(pq)}{\pi(p)}$$

is found to hold true.

A few phenomena are explainable by the property of the probability weighting function of prospect theory (Camerer 2000; Tada 2003). The first is the favorite longshot bias in horse racing. As reported by Thaler and Ziemba (1988), although the expected dividend yield of a long shot with an extremely low probability of winning is much lower than that of betting on the favorite, people are generally willing to bet on a long shot at the races, which is explainable by the characteristics of the probability weighting function that indicates an overvalued probability when the probability is very low. Similarly, the impulses of many people to purchase lottery and lotto tickets are explainable by the characteristics of the probability weighting function.

Furthermore, the reason for numerous people to purchase insurance policies is explainable by the characteristics of the probability weighting function.

Phone line repair insurance, for instance, is 45 cents per month although the repair cost is \$60 and the expected cost of the repair is only 26 cents per month (Chiccheti and Dubin 1994). As this illustrates, the phenomenon of buying insurance coverage is explainable by the interpretation of prospect theory that the weight of an event whose probability is low grows larger.

4 Cumulative Prospect Theory

In the initial paper in 1979, prospect theory was a model to express decision-making under risk (Kahneman and Tversky 1979). In the subsequent paper in 1992, it was expanded to a model that expressed decision-making under uncertainty including ambiguity and risk, which was called cumulative prospect theory (Tversky and Kahneman 1992). Cumulative prospect theory can be interpreted as a type of rank-dependent nonlinear expected utility theory (Quiggin 1993; Starmer 2000; Tamura et al. 1997).

First, the elements of a decision-making problem are defined. Assuming that X is a set of results and assuming that Θ is a set of states of nature, the prospect (alternatives) under uncertainty is defined as $f : \Theta \rightarrow X$. In other words, we consider that such a function that becomes $f(\theta) = x$ exists if $x \in X$ results under a state of nature, as $\theta \in \Theta$. For simplification, however, the result $x \in X$ is assumed to be a monetary value. For example, f is a lottery that gives \$100 (x_1) if a roll of the dice yields an odd number and \$200 (x_2) if an even number comes out.

As preparation for considering cumulative prospect theory, the results are ranked in order of increase in the desirability of the results. For example, the increase in the desirability is arranged from \$100, \$200, and \$400... according to the result. This method of deriving the comprehensive assessment based on the rankings of the desirability of results is basically the same as the case of obtaining the rank-dependent nonlinear expected utility through the Choquet integral (Choquet 1954). In fact, cumulative prospect theory also uses the Choquet integral.

Assuming also that $\{\theta_i\}$ is a subset of Θ and that x_i results if θ_i occurs, the prospect, f, can be expressed with a line of pairs of (x_i, θ_i). The case of rolling the dice described earlier, for instance, can be expressed as Prospect f(\$100; odd number; \$200, even number). In this case, too, the states of nature corresponding to the results are lined up according to the order of desirability of the results.

Cumulative prospect theory assumes that the value function differs between the area of gain and area of loss. Therefore, f^+ as a prospect with a positive result and f^- as a prospect with a negative result are distinguished. Therefore, if $f(\theta) > 0$, then $f^+(\theta) = f(\theta)$; if $f(\theta) \leq 0$, then $f^+(\theta) = 0$; if $f(\theta) < 0$, then $f^-(\theta) = f(\theta)$; and if $f(\theta) \geq 0$, then $f^-(\theta) = 0$. In the earlier case of the dice, $f^+(\theta_1) = \$100$, $f^+(\theta_2) = \$200$, $f^-(\theta_1) = \$0$, and $f^-(\theta_2) = \$0$.

As with expected utility theory, if Prospect f is strictly preferred over Prospect g or is indiscriminate, we consider a function that becomes $v(f) \geq v(g)$ and assume that the comprehensive assessment can be derived with the sum of the functions of the prospect in the area of gain and the prospect in the area of loss.

$$v(f) = v(f^+) + v(f^-)$$

$$v(g) = v(g^+) + v(g^-)$$

Expected utility theory considers an additive set function related to a set of states of nature just as in the case of the body of subjective expected utility of Savage (1954). Cumulative prospect theory, however, considers a non-additive set function from generalized probability measures, which is equivalent to the "capacity" and "fuzzy measure" described in the previous chapter. In other words, it is a set function, $W : 2^\Theta \rightarrow [0,1]$ from an aggregate consisting of subsets of a nonempty set, Θ, of states of nature to a closed interval, [0,1]. Additionally, the boundedness conditions, $W(\phi) = 0$, $W(\Theta) = 1$, and a monotonicity condition (a relation in which when a subset, A_i, of Θ is a subset of A_j, i.e., $A_i \subseteq A_j$, $W(A_i) \leq W(A_j)$) are is satisfied. If, for instance, the degrees of belief that 1, 2, and 3 will be rolled on the dice are 0.1 for each and the degree of belief that odd numbers will be rolled is 0.4, then the additivity condition of probability measures is not satisfied but the monotonicity condition is satisfied.

Cumulative prospect theory assumes that the narrow sense of the monotonically increasing function, $v : X \to Re$ is considered the value function and is normalized to satisfy $v(x_0) = v(0) = 0$. For instance, whereas such a function as $v(x) = 2x^{0.8}$ might be presumed as a specific example, the value function is often discussed in general terms just as when explaining the utility function. The comprehensive assessment, $V(f)$, of the prospect is explained using the sum of $V(f^+)$ and $V(f^-)$ as described earlier, and $V(f^+)$ and $V(f^-)$ are further specified as shown below.

$$V(f) = V(f^+) \text{ and } V(f^-)$$

$$V(f^+) = \sum_{i=0}^{n} \pi_i^+ v(x_i)$$

$$V(f^-) = \sum_{i=-m}^{0} \pi_i^- v(x_i)$$

where $f^+ = (x_0,\ A_0;\ x_1,\ A_1;\ \ldots;\ x_n,\ A_n)$ and $f^- = (x_{-m},\ A_{-m};\ x_{-m+1},\ A_{-m+1};\ \ldots;\ x_0,\ A_0)$ and where $\pi_0^+ \ldots \pi_n^+$ represent the weight in the area of gain and $\pi_{-m}^- \ldots \pi_0^-$ are the weight in the area of loss. Attention must be devoted to the fact that the weight is determined based on the ranking of the desirability of the result.

The weights in cumulative prospect theory are determined as shown below.

$$\pi_n^+ = W^+(A_n),$$

$$\pi_{-m}^- = W^-(A_{-m}),$$

$$\pi_i^+ = W^+(A_i \cup \ldots \cup A_n) - W^+(A_{i+1} \cup \ldots \cup A_n), 0 \leq i \leq n-1,$$

$$\pi_i^- = W^-(A_{-m} \cup \ldots \cup A_i) - W^-(A_{-m} \cup \ldots \cup A_{i-1}), 1-m \leq i \leq 0$$

The above equations are explained briefly in the following. First, the decision-making weight π_i^+ concerns the area of gain, in which the results are positive, and is the difference between the non-additive probability of an event that produces a result that is at least equally desirable as x_i and the non-additive probability of an event that produces a result that is more desirable than x_i. The decision-making weight, π_i^-, concerns negative results and is the difference between the non-additive probability of an event that produces a result that is at least equally desirable as x_i and the non-additive probability of an event that produces a result that is less desirable than x_i. If each W is additive, then W is a probability measure and π_i is simply the probability of A_i.

If the expressions are rewritten as $\pi_i = \pi_i^+$ when $i \geq 0$ and $\pi_i = \pi_i^-$ when $i < 0$ for simplification, then

$$V(f) = \sum_{i=-m}^{n} \pi_i v(x_i)$$

results.

Subsequently, cumulative prospect theory is explainable as follows. If the prospect, $f = (x_i, A_i)$ is given by the probability distribution, $p(A_i) = p_i$, it becomes a problem of decision making under risk, and the prospect can be expressed as $f = (x_i, p_i)$. The decision weight for this problem of decision making under risk is the following.

$$\begin{aligned}
\pi_n^+ &= W^+(p_n), \\
\pi_{-m}^- &= W^-(p_{-m}), \\
\pi_i^+ &= W^+(p_i + \cdots + p_n) - W^+\left(p_{i+1} + \cdots + p_n\right), 0 \leq i \leq n-1, \\
\pi_i^- &= W^-(p_{-m} + \cdots + p_i) - W^-(p_{-m} + \cdots + p_{i-1}), 1-m \leq i \leq 0
\end{aligned}$$

In those equations, W^+ and W^- are monotonically increasing functions in the narrow sense and normalized to $W^+(0) = W^-(0) = 0$ and $(1) = W^-(1) = 1$. As with cumulative prospect theory under uncertainty, the expression that if $i \geq 0$, then $\pi_i = \pi_i^+$ and if $i < 0$, then $\pi_i = \pi_i^-$ results in

$$V(f) = \sum_{i=-m}^{n} \pi_i v(x_i)$$

We consider the following situation to present an example of prospect theory under risk (Tversky and Kahneman 1992). If the number rolled once on the dice is x, then $x = 1, \ldots, 6$. We consider a game in which we gain \$ x if x is an even number and pay \$ x if it is an odd number. Then, f can be thought to be a prospect that produces the results of $(-5, -3, -1, 2, 4, 6)$ with a probability of 1/6 for each result. This inference then engenders the expressions $f^+ = (0, 1/2, 2, 1/6; 4, 1/6, 6, 1/6)$ and $f^- = (-5, 1/6, -3, 1/6; -1, 1/6, 0, 1/2)$ because the probability of \$0 in f^+ is the probability of an odd number, which is 1/2, the probabilities of gaining \$2, \$4, and \$6 are 1/6 each, the probabilities of gaining $-\$5$, $-\$3$, and $-\$1$ in f^- are 1/6 each, and the probability of \$0 is the probability of an even number, which is 1/2. Therefore, the following can be derived.

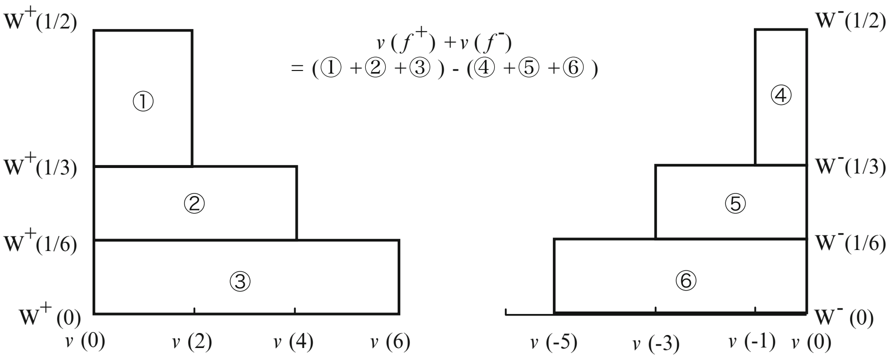


Fig. 8.1 Calculation method of $v(f)$ in cumulative prospect theory. *Note*: $v(f) = v(f^{+}) + v(f^{-})$

$$
\begin{aligned}
V(f) &= V(f^{+}) + V(f^{-}) \\
&= v(2)[W^{+}(1/6+1/6+1/6) - W^{+}(1/6+1/6)] \\
&\quad + v(4)[W^{+}(1/6+1/6) - W^{+}(1/6)] \\
&\quad + v(6)[W^{+}(1/6) - W^{+}(0)] \\
&\quad + v(-5)[W^{-}(1/6) - W^{-}(0)] \\
&\quad + v(-3)[W^{-}(1/6+1/6) - W^{-}(1/6)] \\
&\quad + v(-1)[W^{-}(1/6+1/6+1/6) - W^{-}(1/6+1/6)] \\
&= v(2)[W^{+}(1/2) - W^{+}(1/3)] \\
&\quad + v(4)[W^{+}(1/3) - W^{+}(1/6)] \\
&\quad + v(6)[W^{+}(1/6) - W^{+}(0)] \\
&\quad + v(-5)[W^{-}(1/6) - W^{-}(0)] \\
&\quad + v(-3)[W^{-}(1/3) - W^{-}(1/6)] \\
&\quad + v(-1)[W^{-}(1/2) - W^{-}(1/3)]
\end{aligned}
$$

This relation is expressed in Fig. 8.1. The $V(f^{+})$ is the area on the left-hand side of Fig. 8.1, and $V(f^{-})$ is the product of the area on the right-hand side of Fig. 8.1 multiplied by -1. When this is expressed linguistically, the comprehensive assessment in cumulative prospect theory can be derived as follows: First, the weight of the value of \$2, π, is obtained from the difference between the weight, w, of the probability of gaining \$2 or more and the weight, w, of the probability of gaining \$4 or more. Other weights π are obtainable in the same manner. The sum of products of this π and the value v engenders the comprehensive assessment.

5 Experiments on Cumulative Prospect Theory

Tversky and Kahneman (1992) conducted selection experiments by presenting various prospects on computers to a total of 25 graduate students at Stanford and Berkeley and estimated the value function of cumulative prospect theory. They presented such prospects as the probability of gaining $150 was 25% and the probability of gaining $50 was 75%. They also compared such prospects with definite prospects and conducted experiments of selecting desirable ones. They assumed the following power functions as value functions.

$$v(x) = x^{\alpha} \ (when\, x \ \geq \ 0)$$

$$v(x) = -\lambda\left(-x^{\beta}\right) \ (when\, x < 0)$$

Based on the results of the selection in this experiment, they performed a nonlinear regression analysis and estimated 0.88 for both α and β and 2.55 for λ. The fact that the estimated values of α and β are less than 1 indicates that the value function is concave downward in the area of gain and convex downward in the area of loss. The estimated value of λ suggests that the loss has an impact that is approximately twice as great as that of a profit, implying the strong nature of loss aversion.

They further considered the following functions as specific decision weight functions, W^{+} and W^{-}, of cumulative prospect theory, and estimated the form of the decision weight functions illustrated in Fig. 8.2 from this selection experiment.

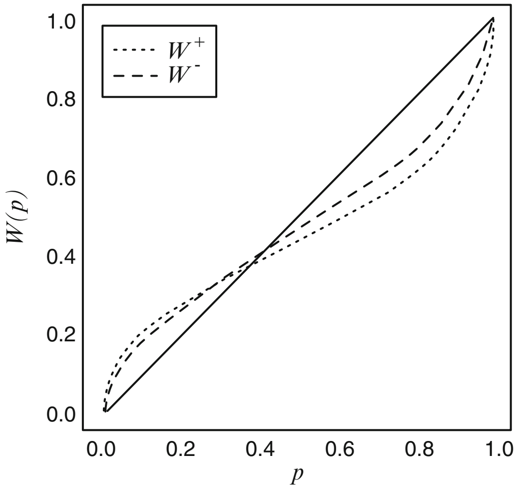


Fig. 8.2 Estimated weighted probabilities. *Note*: W^{+} indicates weighted probability function for gain, and W^{-} indicates weighted probability function for loss. *Source*: Tversky and Kahneman (1992)

$$W^{+}(p) = \frac{p^{\gamma}}{\left(p^{\gamma} + (1-p)^{\gamma}\right)^{\frac{1}{\gamma}}}, \quad W^{-}(p) = \frac{p^{\delta}}{\left(p^{\delta} + (1-p)^{\delta}\right)^{\frac{1}{\delta}}}$$

The estimated value of γ is 0.61 and the value of δ is 0.69. The value of δ is slightly greater than the value of γ as depicted in Fig. 8.2, suggesting that the curve of the probability weighting function for positive results is slightly sharper than that for negative results.

References

Benartzi, S., & Thaler, R. H. (1995). Myopic loss aversion and the equity premium puzzle. *Quarterly Journal of Economics, 110*, 73–92.

Camerer, C. F. (2000). Prospect theory in the wild: Evidence from the field. In D. Kahneman & A. Tversky (Eds.), *Choices, values, and frames* (pp. 288–300). New York: Cambridge University Press.

Chiccheti, C., & Dubin, J. (1994). A microeconometric analysis of risk aversion and the self-insure. *Journal of Political Economy, 102*, 169–186.

Choquet, G. (1954). Theory of capacities. *Annales de l' Institute Fourier, 5*, 131–295.

Kahneman, D., Knetsch, J. L., & Thaler, R. H. (1990). Experiment tests of the endowment effect and coase theorem. *Journal of Political Economy, 98*, 1325–1348.

Kahneman, D., Knetsch, J. L., & Thaler, R. H. (1991). The endowment effect, loss aversion, and status quo bias. *Journal of Economic Perspectives, 5*, 193–206.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47*, 263–292.

Odean, T. (1998). Are investors reluctant to realize their losses? *Journal of Finance, 53*, 1775–1798.

Quiggin, J. (1993). *Generalized expected utility theory: The rank dependent model*. Boston: Kluwer Academic Publishers.

Savage, L. J. (1954). *The foundations of statistics*. New York: John (Wiley) & Sons.

Shefrin, H. M., & Statman, M. (1985). The disposition to sell winners too early and ride losers too long. *Journal of Finance, 40*, 777–790.

Starmer, C. (2000). Developments in non-expected utility theory: The hunt for descriptive theory of choice under risk. *Journal of Economic Literature, 38*, 332–382.

Tada, Y. (2003). *Kodokeizaigaku Nyumon* (*Introduction to behavioral economics*). Tokyo: Nikkei Inc.

Tamura, H., Nakamura, Y., & Fujita, S. (1997). *Koyo Bunseki no Suri to Oyo (Mathematical principles and application of utility analysis)*. Tokyo: Corona Publishing.

Thaler, R. H., & Ziemba, W. T. (1988). Parimutuel betting markets: Racetracks and lotteries. *Journal of Economic Perspectives, 2*, 161–174.

Toshino, M. (2004). *Shoken shijo to kodo fainansu (Security market and behavioral finance)*. Tokyo: Toyo Keizai.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5*, 297–323.

Part V
The Framing Effect and Its Descriptions

Chapter 9
The Framing Effect of Decision Making

The body of utility theory that explains the phenomenon of decision making has developed new theories such as nonlinear utility theory to explain decision making under uncertainty. There is, however, a phenomenon called the framing effect which cannot be explained in its essence by the body of utility theory. The framing effect refers to phenomena in which preference is reversed even for the same decision making problem because of changes in perspectives resulting from differences in the linguistic expressions used to describe the decision making problem, resulting in varied results of decision making.

This chapter reports the findings of studies of behavioral decision-making theory, including the reasons underlying the inability of utility theory to explain the framing effect, the observed intensity of the framing effect, and the types of factors affecting the framing effect.

1 What Type of Phenomenon Is the Framing Effect?

The framing effect can be interpreted as a phenomenon in which the results of decision making vary because of differences in the mental framework of understanding the decision making problem, i.e., the decision frame (Tversky and Kahneman 1981).

We assume the following situation. A tumor was discovered in your lungs during a medical checkup and your doctor has recommended that you have a surgery. Your willingness to have the surgery is likely to vary substantially in the cases when your doctor tells you, “I have performed the surgery on 1,000 patients in the past, of whom 950 patients lived for more than five years. Why don’t you have the surgery, too?” and when he says, “I have performed the surgery on 1,000 patients in the past, of whom 50 patients died within five years. Why don’t you have the surgery, too?” (Takemura 1996). The former expression presents a decision frame that emphasizes survival, although the latter expression is a decision frame that emphasizes death. The information provided by the former linguistic expression, rather than the latter,

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_9,

is likely to encourage more people to opt to undergo the surgery. The phenomenon in which such a difference arises in the results of a decision is designated as the framing effect.

Tversky and Kahneman (1981) were the first to study this framing effect systematically. They assumed the following decision making problem, which would be an example of the framing effect research, for their investigation.

The problem of an Asian disease: “We assume that a peculiar Asian disease that is expected to kill 600 people in the U.S. has broken out. Measures of two types were proposed to cure this disease. The precise scientific estimates of these measures are presented below. Which one of the measures would you adopt?”

They first divided 307 college students into two groups and presented the alternatives using the following expressions to 152 students for the frame conditions that emphasize survival (positive frame conditions).

Measure A: “Two hundred people will be saved if this measure is adopted.”
Measure B: “The probability that 600 people will be saved is one-third and the probability that no one will be saved is two thirds if this measure is adopted.”

As a result, 72 % selected Measure A and 28 % opted for Measure B.

Meanwhile, they presented the following measures—which are frame conditions for the same decision making problem but expressed in terms of mortality (negative frame conditions)—to the remaining 155 students.

Measure C: “Four hundred people will die if this measure is adopted.”
Measure D: “The probability that no one will die is one-third and the probability that 600 people will die is two thirds if this measure is adopted.”

Measures A and C and Measures B and D differ only in their expressions. The extensionally recognizable meanings should be same despite the different expressions. In other words, “being saved” is equivalent to “not dying” and “not being saved” means “to die.” Nevertheless, those students who selected Measure C comprised 22 % whereas those who chose Measure D were 78 %. This reversal of selection patterns indicates the framing effect. Tversky and Kahneman (1981) have reported that most test subjects use risk-averse selection (A = C) when the aspect of benefit is emphasized in the expression such as positive frame conditions and risk-taking selection (D = B) when the aspect of the loss in the decision problem is emphasized in the expression such as negative frame conditions.

2 Why Can Utility Theory Not Explain the Framing Effect?

One of the most representative theories of decision making is utility theory, which attempts to explain the overall decision making phenomena based on the concept of utility. This utility theory dates back to Bernoulli in the 18th century. Although numerous variations have been developed (Fishburn 1982, 1988; Schmeidler 1989;

Starmer 2000), it is a mathematical model based on the subjects defined by factors that allow objective observation and instruction (extensionally defined subjects) as do mathematical models in the natural sciences.

The existence of the framing effect means that different decisions are made even for extensionally identical decision problems, which deviates from the principle of description invariance (Arrow 1982) that the theoretical conclusion is the same if the subject is extensionally defined. Description invariance is therefore a principle that demands no changes in the result resulting from the manner of speech and description. Consequently, the framing effect cannot be explained in essence by utility theory that assumes description invariance, which is explained more specifically in the following using the example of decision making under certainty.

Imagine the case of decision making under certainty explained below (Takemura 1994b). Consider making the decision of selecting either fried rice or Tenshindon (crab meat omelet on rice) at a Chinese restaurant. In this case, the utility refers to the value of the real number that makes the utility of Tenshindon (u(Tenshindon)) greater than or equal to the utility of fried rice (u(fried rice)) when and only when preferring fried rice to Tenshindon or equally liking both (fried rice $\succsim$ Tenshindon). Therefore, the relation,

$$\text{fried rice} \succsim \text{Tenshindon} \Leftrightarrow u(\text{fried rice}) \geq u(\text{Tenshindon}),$$

holds true. We assume, at this point, that a “Tenshindon” refers in this Chinese restaurant to a “Chinese-style egg bowl” and a “Chinese-style egg bowl” refers to a “Tenshindon.” Then, according to description invariance, both the relations

$$\text{fried rice} \succsim \text{Tenshindon} \Leftrightarrow \text{fried rice} \succsim \text{Chinese-style egg bowl}$$

$$\text{and } u(\text{fried rice}) \geq u(\text{Tenshindon}) \Leftrightarrow u(\text{fried rice}) \geq u(\text{Chinese-style egg bowl})$$

must be valid.

If the framing effect occurs and these relations cannot be established in such a case as

$$\text{fried rice} \succsim \text{Tenshindon} \Leftrightarrow \text{fried rice} \prec \text{Chinese-style egg bowl},$$

then the relation becomes

$$u(\text{fried rice}) \geq u(\text{Tenshindon}) \Leftrightarrow u(\text{fried rice}) < u(\text{Chinese-style egg bowl})$$

according to the definition of utility, which does not satisfy description invariance. Consequently, if fried rice is preferred when the other dish is called Tenshindon but the same dish is preferred to fried rice when it is called a Chinese-style egg bowl, then neither of the preference relation or utility function satisfies description invariance. Utility theory that assumes description invariance therefore cannot explain the framing effect. This inability to explain is also true, in general, not only in decision-making under certainty presented here, but also in nonlinear utility theory under uncertainty. Therefore, the point underscores the inability of utility theory to explain the framing effect.

Furthermore, the framing effect cannot be explained by any theory that assumes description invariance in addition to utility theory, which suggests the difficulty of explaining the framing effect using an extensionally definable set. Because of this, the presence of the framing effect brings us the question of "what is a meaning?" The same problem was introduced by Frege, the founder of the predicate logic, who noted that "the morning star" signifies Venus just as "the evening star" does. Therefore, the situation of two utterances having the same meaning (Bedeutung), but having different significance (Sinn) lies under the purview of the problem of framing. Although the framing effect appears to be very difficult to treat in a theoretical manner, as shown up to this point, the verification of the effects and their explanation using approximate descriptive theories have been achieved in various ways.

3 Framing in Social Life

The framing effect might be observed in various scenes in social life. This framing effect has long been recognized in the world of market research professionals as the wording effect of a slight change in the linguistic expressions of the questions that would change the responses to the questions. It had been long known also in the world of marketing professionals as the promotion effect of a variation in the linguistic expressions used in advertisements: despite providing the same product information, different linguistic expressions can drastically alter effects on sales. In consumer behavior research, Kojima (1986) addressed the phenomenon that spending attitudes vary depending on the context even for the same commodity. Kojima referred to the effect of a "psychological wallet," which is equivalent to the framing effect.

The framing effect can be detected easily first in consumers' decision making. It is no secret that, in communication activities such as advertising and sales, subtle differences in expressions having the same meaning in messages delivered to consumers alter shoppers' assessments, judgments, and purchase decisions.

Ground meat, for instance, is divisible into lean meat and fatty meat. We know that consumers tend to assess the meat more favorably when it is labeled "75 % lean meat" than when it is labeled "25 % fatty meat" (Levin and Gaeth 1988). The framing effect can also be observed when making a decision to purchase auto insurance. An example is a case of an insurance policy for which the premium is $1,000 with a $600 deductible on each claim settlement and another insurance policy for which the premium is $1,600 that covers damage of $600 and offers $600 cash back to a policyholder with no accidents. Although both policies are ultimately equivalent, we know that the latter is preferred by more people (Johnson et al. 1993). In recent years, foreign-owned insurance companies have been advertising such insurance policies as the latter that offer cash back, which is also regarded as using the framing effect of consumers. Additionally, the framing effect has been identified in physicians' medical decision making (McNeil et al. 1982) and business managers' decision making (Qualls and Puto 1989).

The effect can be further detected in interpersonal interactions. Kelly and Thibaut (1978) assumed the profit matrix in their theory of interpersonal dependency as a given matrix in an experiment game and concluded that people would psychologically convert this matrix to an effective matrix before making a decision. They pointed out that the conversion from the given matrix to an effective matrix in interpersonal relationships included various types. The problem of converting from the given matrix to an effective matrix argued by them can be regarded as a problem of framing in interpersonal decision making.

The mode of framing by decision makers has been found to affect decision-making significantly also in experiment games (Colman 1995). Eiser and Bhavnani (1974), for example, examined how decision making would vary as a result of changes in the instructions for the situation of prisoner's dilemma games with identical structure. They instructed the test subjects under each set of conditions on the situation setting as problems of economic negotiations, international negotiations, and human relationships, and the game counterpart (a decoy) used a retribution strategy under all conditions in the experiments. The reported result was that the percentage of participants selecting cooperation to achieve Pareto optimality was higher when the test subjects were instructed as a problem of international negotiations or human relationships rather than the problem of economic negotiations. This experimentally obtained result is thought to indicate the framing effect.

The problem of framing is extremely important when considering decision-making in disputes, which is particularly clarified when considering the issue of international disarmament (Tversky 1994). We assume that two countries are negotiating the reduction of the number of missiles. Reducing the missiles of one's own country is perceived as a loss from the current conditions and reducing the missiles of the other country is viewed as a benefit. According to Tversky (1994), the impact of a perceived loss is approximately twice the impact of a perceived benefit. Therefore, eliminating two missiles of the other country and eliminating one missile of one's own country presumably balances out. Agreement between the two countries is therefore extremely difficult because both negotiators could be expected to regard the problem similarly. Although the idea of losing or gaining from the current position might be acceptable in general, how the current position is understood depends substantially on the framing of the situation of the problem.

4 Framing and Mental Accounting

This framing effect is interpreted as resulting from mental structuring of a decision frame based on the linguistic expressions of a decision making problem and other factors. How, then, is such a decision frame structured? Particularly in decision making related to money, Tversky and Kahneman (1981) used the concept of so-called mental accounting to explain how the decision frame was structured. Mental accounting refers to the form used by people for cognitive processing of monetary decision making problems, which is the state of "psychological wallet" introduced by Kojima (1986).

4.1 Losing a Ticket and Mental Accounting

Tversky and Kahneman (1981) asked the following questions to 383 test subjects and thereby examined the state of mental accounting. They asked 200 test subjects the following question under the conditions of losing a ticket.

Conditions of losing a ticket: "Imagine the following scenario. You have decided to go to see a movie and go to the theater after purchasing a ticket priced at $10. When you are about to enter the theater, you notice that you have lost the ticket. Would you buy another ticket?"

They asked the remaining 183 test subjects the following question under the conditions of losing cash.

Conditions of losing cash: "Imagine the following scenario. You have decided to go to see a movie and go to the theater. The ticket is priced at $10. When you are about to enter the theater, you notice that you have lost the $10 in cash. Would you buy the ticket?"

As a result of the questions, although 46 % of the test subjects under the conditions of losing the ticket answered that they would buy another ticket, 88 % of the test subjects under the conditions of losing the cash answered that they would buy the ticket.

The point requiring attention here is the fact that, in both cases, the loss is equivalent to $10 and the decision of whether to buy a ticket that is worth $10 must be made. Tversky and Kahneman (1981) explain that the difference in the results is attributable to the varied state of mental accounting between the conditions of losing a ticket and losing cash. In other words, this can be interpreted as a situation in which, although the conditions of losing the ticket require a purchase of another ticket from the account of ticket expenditure (a type of psychological wallet), the conditions of losing the cash do not cause the pain of buying two tickets because the cash and ticket expenditures are in separate accounts, resulting in greater willingness to purchase the ticket. Only the ticket account is used when purchasing the ticket. Therefore, the loss of cash is regarded as having a moderate effect.

Consequently, Tversky and Kahneman (1981) conclude that mental accounting tends to be conducted for each topic rather than constituting a comprehensive monetary assessment.

4.2 Purchasing a Calculator and Mental Accounting

Tversky and Kahneman (1981) also asked 192 test subjects the following question on the mental accounting of consumers. They asked 88 test subjects the question below related to conditions surrounding the purchase of a $15 calculator.

Conditions of a $15 calculator: "Imagine the following situation. You are going to purchase a $125 jacket and $15 calculator, but the store attendant tells you that the $15 calculator is sold for $10 at a branch store that is 20 min by car from this store. Would you go to the branch store to buy the calculator?"

They asked the remaining 93 test subjects the question below under the conditions of a \$125 calculator.

Conditions of a \$125 calculator: "Imagine the following situation. You are going to purchase a \$125 calculator and \$15 jacket, but the store attendant tells you that the \$125 calculator is sold for \$120 at a branch store that is 20 min by car from this store. Would you go to the branch store to buy the calculator?"

The two sets of conditions are common as a case of making a decision to purchase a calculator and jacket and identical in the question of whether to do the shopping of \$140 in all or go to the branch to save \$5 at the cost of driving a car for 20 min. As a result of the questions, although 68 % of the test subjects under the former conditions of the \$15 calculator answered that they would go to the branch store, 29 % of the test subjects under the latter conditions of the \$125 calculator answered that they would go to the branch store.

This result might be attributable to the fact that the test subjects framed the two decision-making problems separately rather than considering the purchase of the calculator and the purchase of the jacket as a combination, which is another indication that the state of mental accounting is not comprehensive; rather, each topic is processed separately. Recognizing the problem as the question of whether to do the shopping of \$140 or \$135 in all should make the assessment results of the two sets of conditions identical. In the conditions of the \$15 calculator, however, the fact that the calculator whose list price is \$15 becomes \$10 is presumably emphasized; in the conditions of the \$125 calculator, the fact that the calculator's list price, \$125, becomes \$120 is emphasized. The cost reduction from the calculator's list price of \$15 to \$10 would be valued more than the cost reduction from \$125 to \$120 if a negative utility function that is convex downward was assumed, just as in prospect theory.

4.3 Hedonic Framing

Thaler (1985, 1999) argues that the state of mental accounting is based on the principle of hedonic framing, in which diverse factors of decision making problems are separated and combined to achieve higher total assessments. Assuming two factors, x and y, and assuming that $x \circ y$ is the combination of x and y, he argues that hedonic framing is conducted according to such a rule as

$$v(x \circ y) = \max(v(x + y), v(x) + v(y)).$$

Based on the assumption of the value function of prospect theory, he points out that the characteristics described below can be found in hedonic framing.

1. Gains are framed by separating each topic (because the value function of a gain is concave downward, the total assessment becomes higher when separated).
2. Losses are framed by combining various topics (because the value function of a loss is convex downward, the total assessment becomes higher when combined).

3. Minor losses and major gains are combined and framed (loss aversion is offset).
4. Minor gains and major losses are separated and framed (because the slope of the value function in the area of gain is steep near the origin, slightly increasing a gain has a greater impact than slightly reducing a major loss).

According to the principle of hedonic framing of Thaler (1985, 1999), a gain to consumers such as that of a discount tends to be separated and framed. As suggested by the results of the questions related to the calculators asked by Tversky and Kahneman (1981), if discounts are separated for each product and framed, when a consumer intends to purchase two or more products, then increasing the discount on a lower-priced product is expected to have a greater marketing effect than increasing the discount on a higher-priced product.

Supermarkets, for instance, often use the strategy of collecting customers by offering a large discount on a product such as a pack of eggs—reduced from the normal price of $2.50 to $1.00—without substantially reducing the prices of high-priced products, which increases the unit price of all purchased products as a whole (egg prices are stable, making it easier for consumers to recognize discounts). The $1.50 discount on the eggs is expected to have a greater effect even if a high-priced product such as a television is discounted by $5.00 or more if the supermarkets provide consumers with a total discount equivalent to or greater than the discount on a pack of eggs.

Accordingly, using mental accounting by understanding its characteristics helps develop strategies that are meaningful in the sense of marketing. It also allows consumers to avoid manipulation by companies.

5 Robustness of the Framing Effect

Tversky and Kahneman (1981, 1986) state that the framing effect in decision-making is a very robust phenomenon. Contradictory judgments are made in the process, which might be noticed after the fact, just as the phenomenon of optical illusion in perception.

They speak to the difficulty that, like the Muller–Lyer illusion in psychology, the framing effect is extremely prone to biases that are difficult to eliminate even if detected (Tversky and Kahneman 1986). The Muller–Lyer illusion is a strong phenomenon presented in Fig. 9.1: when arrow feathers are attached to both ends of lines of equal length, inward attachment makes the line look shorter (upper panel) although outward attachment makes the line appear to be longer (lower panel).

The framing effect can also be compared to the Ebbinghaus areal illusion. This optical illusion is a phenomenon by which, as portrayed in Fig. 9.2, a middle-sized circle appears smaller when surrounded by multiple large circles (upper panel) and larger when surrounded by multiple small circles (lower panel). In both optical illusions, a small amount of contextual information makes the subjects of the same size appear differently. Tversky and Kahneman (1981, 1986) report that the framing effect has a function that is similar to such phenomena of optical illusions.

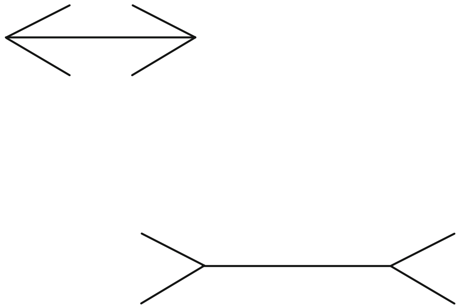


Fig. 9.1 Müller–Lyer illusion

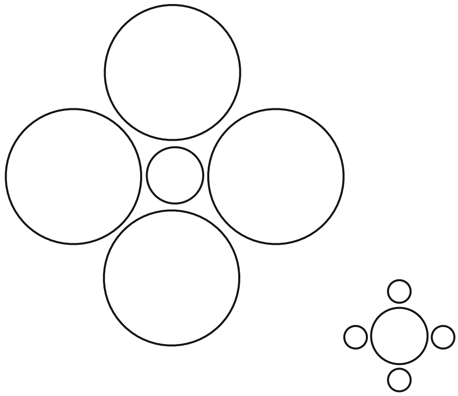


Fig. 9.2 Ebbinghaus illusion

We are unable in most cases to correct our contradictions in decision making if the framing effect is a robust phenomenon, as they argue. Nonetheless, in contrast to their claim, there have been cases in which the framing effect is not observed (Fagley and Miller 1987; Rybash and Roodin 1989). In addition, Takemura (1992, 1993, 1994a) has found that the framing effect is controlled by such operations as prolonging the time required for decision-making or justifying a decision before making a decision.

Takemura (1994b) examined the following hypothesis in psychological experiments. This hypothesis forecasts that while the framing effect is observed under the conditions that require no justification of a decision prior to decision making, it is not observed under conditions requiring justification, in which the test subjects are to write the reasons for their decisions in a form.

The test subjects were 180 male and female college students, and the problem of the Asian disease in Tversky and Kahneman (1981) was used as the subject of decision making (cf. p.124). The test subjects are assigned randomly to one of two (justify decision or not) times two (decision frames: positive and negative) groups.

The results, presented in Table 9.1, support the hypothesis. The framing effect was detected under the conditions without the procedure of justification. In other words, 80.0 % of the test subjects made a risk-free selection under positive frame conditions; conversely, however, 68.9 % of the test subjects made a risky selection under the negative frame conditions. Meanwhile, no statistically significant framing effect was identified under the conditions with the procedure of justification. Therefore, 46.7 % of the test subjects made a risk-free selection under the positive frame conditions and 37.8 % of the test subjects made a risky selection under the negative frame conditions.

Table 9.1 Framing effect in justification condition and no justification condition

	Justification condition		No justification condition	
Frame condition	Positive	Negative	Positive	Negative
Riskless option	36	14	21	28
Risky option	9	31	24	17

Source: Takemura (1994a)

Such experimentally obtained results presented by Takemura (1994b) are made to question the robustness of the framing effect, suggesting that the effect might be eliminated by such operations as cognitive elaboration. In this connection, Fujii and Takemura (2001) report that the framing effect can be controlled by manipulating attention using the size of literal information. Kuhberger (1998) meta-analyzed numerous experimental studies of the framing effect made in the past and reported that the framing effect was controlled in some cases by response modes and characteristics of decision problems.

The controversy surrounding the robustness of the framing effect has continued in recent years. An experiment has been reported as showing that the framing effect appears robustly, as asserted by Tversky and Kahneman (1981, 1986), which is not eliminated by the procedure of justification, irrespective of whether the need for cognition of the test subjects is high or low (LeBoeuf and Shafir 2003). In contrast to this report, Simon et al. (2004) produced findings from experiments and studies that question this robustness. They present that the framing effect does not disappear when those with a low need for cognition go through the procedure of justification. However, those with a high need for cognition related to the form of analytical thinking are able to eliminate the framing effect through the process of justifying their decisions.

Consequently, although no clear conclusion related to the robustness of the framing effect has been identified, at least it has been found not to be as robust as Tversky and Kahneman (1981, 1986) initially surmised. Considering, however, that we do not often engage ourselves in cognitive elaboration in our normal social life (e.g., Langer 1978), the framing effect is expected to occur almost constantly in decision making in our social life.

References

Arrow, K. J. (1982). Risk perception in psychology and economics. *Economic Inquiry, 20*, 1–9.

Colman, A. M. (1995). *Game theory and its applications in the social and biological sciences*. Oxford: Butterworth–Heinemann.

Eiser, J. R., & Bhavnani, K. K. (1974). The effect of situational meaning on the behavior of subjects in the prisoner's dilemma game. *European Journal of Social Psychology, 4*, 93–97.

Fagley, N. S., & Miller, P. M. (1987). The effects of decision framing on choice of risky vs. certain options. *Organizational Behavior and Human Decision Processes, 39*, 264–277.

Fishburn, P. C. (1982). *The foundations of expected utility*. Dordrecht: D. Reidel.

Fishburn, P. C. (1988). *Nonlinear preference and utility theory*. Baltimore: The Johns Hopkins University Press.

Fujii, S., & Takemura, K. (2001). Risuku taido to chui: Jokyo izonteki shoten moderu ni yoru furemingu koka no keiryo bunseki [Risk attitude and attention: A psychometric analysis of framing effect by contingent focus model]. *Kodo keiryogaku [The Japanese Journal of Behaviormetrics], 28*, 9–17.

Johnson, E. J., Hershey, J., Meszaros, J., & Kunreuther, H. (1993). Framing, probability distortions, and insurance decisions. *Journal of Risk and Uncertainty, 7*, 35–51.

Kelly, H. H., & Thibaut, J. W. (1978). *Interpersonal relations: A theory of interdependence*. New York: Wiley.

Kojima, S. (1986). *Kakaku no Shinri: Shohisha ha Nani wo Konyu Kettei no Monosashi ni Surunoka [Price psychology: What measure do consumers use to make purchasing decisions?]*. Tokyo: Diamond.

Kuhberger, A. (1998). The influence of framing on risky decisions: A meta-analysis. *Organizational Behavior and Human Decision Processes, 75*, 23–55.

Langer, E. J. (1978). Rethinking the role of thought in social interaction. In J. H. Harvey, W. Ickes, & R. F. Kidd (Eds.), *New directions in attribution research* (Vol. 2, pp. 35–58). Hillsdale: Lawrence Erlbaum Associates.

LeBoeuf, R. A., & Shafir, E. (2003). Deep thoughts and shallow frames: On the susceptibility to framing effects. *Journal of Behavioral Decision Making, 16*, 77–92.

Levin, I., & Gaeth, G. J. (1988). How consumers are affected by the framing of attribute information before and after consuming the product. *Journal of Consumer Research, 15*, 374–378.

McNeil, B. J., Pauker, S. G., Sox, H. C., & Tversky, A. (1982). On the elicitation of preferences for alternative therapies. *The New England Journal of Medicine, 27*, 1259–1262.

Qualls, W. J., & Puto, C. P. (1989). Organizational climate and decision framing: An integrated approach to analyzing industrial buying decisions. *Journal of Marketing Research, 26*, 179–192.

Rybash, J. M., & Roodin, P. A. (1989). The framing heuristic influences judgments about younger and older adults' decision to refuse medical treatment. *Applied Cognitive Psychology, 3*, 171–180.

Schmeidler, D. (1989). Subjective probability and expected utility without additivity. *Econometrica, 57*, 571–587.

Simon, A. F., Fagley, N. S., & Halleran, J. G. (2004). Decision framing: Moderating effects of individual differences and cognitive processing. *Journal of Behavioral Decision Making, 17*, 77–93.

Starmer, C. (2000). Developments in non-expected utility theory: The hunt for descriptive theory of choice under risk. *Journal of Economic Literature, 38*, 332–382.

Takemura, K. (1992). Effect of decision time on framing of decision: A case of risky choice behavior. *Psychologia, 35*, 180–185.

Takemura, K. (1993). The effect of decision frame and decision justification on risky choice. *Japanese Psychological Research, 35*, 36–40.

Takemura, K. (1994a). Influence of elaboration on the framing effect. *Journal of Psychology, 128*, 33–39.

Takemura, K. (1994b). Furemingu Koka no Rironteki Setsumei: Risukuka Deno Ishikettei no Jokyo Izonteki Shoten Moderu [Theoretical explanation of the framing effect: Contingent focus model for decision-making under risk]. *Japanese Psychological Review, 37*, 270–293.

Takemura, K. (1996). *Ishikettei no Shinri: Sono Katei no Tankyu [Mentality of decision making: The quest for the process]*. Tokyo: Fukumura Shuppan.

Thaler, R. H. (1985). Mental accounting and consumer choice. *Marketing Science, 4*, 199–214.

Thaler, R. H. (1999). Mental accounting matters. *Journal of Behavioral Decision Making, 12*, 183–206.

Tversky, A. (1994). Contingent preferences: Loss aversion and tradeoff contrast in decision making. *Japanese Psychological Research, 36*, 3–9.

Tversky, A., & Kahneman, D. (1981). The framing of decisions and the psychology of choice. *Science, 211*, 453–458.

Tversky, A., & Kahneman, D. (1986). Rational choice and the framing of decisions. *Journal of Business, 59*, 251–278.

Chapter 10
Theories Used to Explain the Framing Effect

Chapter 9 introduced empirical studies of the framing effect. This chapter will explain the reasons why the framing effect occurs. Tversky and Kahneman (1981) explained the framing effect based on prospect theory (Kahneman and Tversky 1979, Tversky and Kahneman 1992), which they proposed. This chapter will first use this prospect theory to explain the framing effect, and subsequently use the contingent focus model (Takemura 1994; Takemura and Fujii 1999, Fujii and Takemura 2001a)—an alternative explanation to prospect theory—for the explanation and introduce the empirical research related to this model.

1 Using Prospect Theory to Explain the Framing Effect

How does the framing effect occur? According to prospect theory proposed by Tversky and Kahneman (1981), the decision making process is divided into the editing phase, in which the problem is recognized, and the evaluation phase, in which alternatives are evaluated based on recognition of the problem (see Fig. 10.1). During the editing phase, decision-related problems, despite having the identical objective characteristics, entail different recognition depending on framing that is altered by even a slight difference in linguistic expressions.

Prospect theory defines a result as a deviation from the reference point, constituting the psychological origin. The decision maker is thought to evaluate the result as either a gain or loss. Figure 10.2 presents the framing effect of the problem of the disease in Asia described in the preceding chapter using the value function of prospect theory. The value function is a concave function in the area of gain. It signifies risk-averse behavior, and is a convex function in the area of loss, indicating risk-taking behavior. Moreover, Fig. 10.2 shows that the slope of the value function is generally steeper in the area of loss than it is in the area of gain. A peculiarity of prospect theory is that the reference point corresponds to the origin in utility theory, which is assumed to shift easily depending on how to frame the decision problem, which explains that such a shift of the reference point causes

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_10,

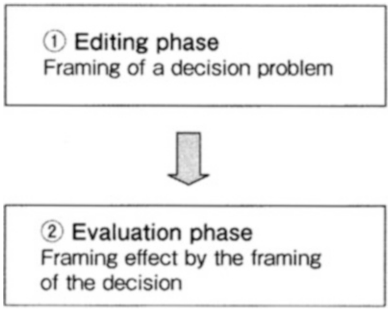
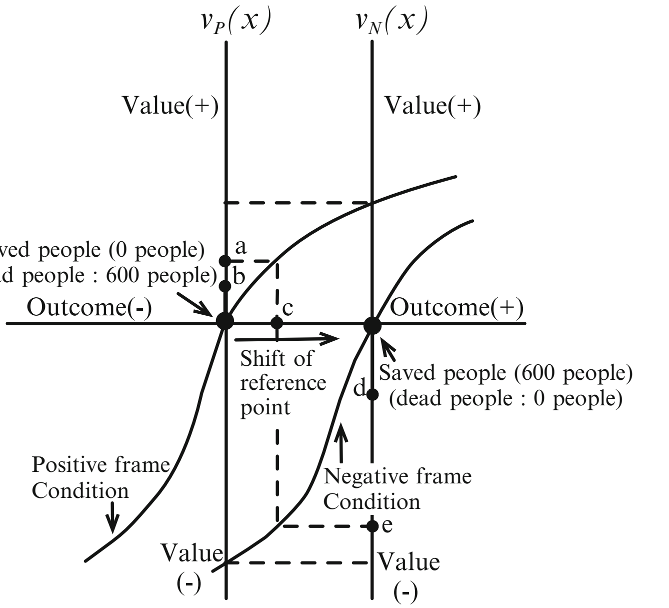


Fig. 10.1 Psychological process in framing effect by prospect theory



a : Value for the program with 200 saved people.
b : Value for the program with 600 saved people probability, 1/3.
c : 200 saved people (400 dead people).
d : Value for the program with 600 saved people probability, 1/3.
e : Value for the program with 400 dead people.

Fig. 10.2 Interpretation of framing effect in Asian disease problem by prospect theory

risk-averse behavior under positive frame conditions and risk-taking behavior under negative frame conditions, even for an identical decision-related problem. More specifically, the decision maker evaluates the result that 200 people survive under the positive frame condition as a gain using the concave function and same result that 400 people die under the negative frame condition as a loss using the convex function.

In addition, Tversky and Kahneman (1981) pointed out, as another cause of the framing effect, that the size of the load of probabilities on preference has a nonlinear relation. Therefore, the value of the definite gain or loss from an alternative would be greater, thereby increasing the framing effect.

These ideas of prospect theory explain why the description invariance by which formally equivalent decision making problems that bring about the same preference order cannot be satisfied. Although the difference in the characteristics of the value functions between the areas of gain and loss and the non-additive effect of probabilities on preference have been adopted in many other nonlinear utility theories, the essence of prospect theory is that preference is reversed by a shift of the reference point.

2 Problems of Prospect Theory

The presence of the reference point is extremely important for explanation of the framing effect based on prospect theory. What type of formulation can then provide a theoretical explanation of a shift of the reference point? Tversky and Kahneman (1981) presented the following opinions in this regard. “The frame used by the decision maker relies on the form of the selection problem or the norms, customs, or personal characteristics of the decision maker (Ref. p. 453).” However, they have merely presented such a qualitative remark and fail to furnish the reader with a clear answer up to the present. Currently, prospect theory fails to clarify how the reference point changes and has difficulty in forecasting preference and selection. In other words, although assuming that the decision maker converts the coordinate system, prospect theory has a problem of not indicating how the decision maker converts the coordinate system (Takemura 1994; Fujii and Takemura 2001a).

On the assumption that prospect theory is correct, Fischhoff (1983) attempted to identify the position of the reference point theoretically from the result of selection. For many test subjects, however, the position of the reference point was not identified successfully. Furthermore, Fischhoff discovered in the responses of many of the test subjects that the value of the reference point as reported by a test subject in person after the test was not consistent with the reference point inferred from the result of selection.

In addition, although prospect theory assumes only one reference point, the reference point of decision making is not necessarily only one. The possibility exists that a decision maker uses multiple reference points (Takemura 1998, 2001). In fact, in the analysis of the linguistic reports on the test subjects in the problem of the Asian disease, slightly more than 40 % of the test subjects (5 out of 12) were found to have made decisions based on multiple reference points in at least one of the two decision problems (Maule 1989).

Prospect theory is therefore difficult to use as a theory for quantitative description of behavioral decision making (Fujii and Takemura 2001a) for the two reasons of the problem of identifying the reference point (Fischhoff 1983) and the problem of the possible presence of multiple reference points (Maule 1989; Takemura 1998, 2001).

3 Basic Assumptions of the Contingent Focus Model as an Alternative Explanation

In an effort to explain the framing effect theoretically without using the concept of the "reference point", which presents quantitative problems, Takemura and Fujii proposed a model—the contingent focus model—that assumes dependence of the framing effect on the attention of the decision maker (Takemura 1994; Takemura and Fujii 1999, Fujii and Takemura 2001a). This model includes the assumption that an increase in the weight of attributes being focused upon and changes in the attributes focused upon by the decision maker are caused by linguistic expressions.

In this model, the framing effect is regarded as occurring not necessarily because of changes in the reference point as claimed by prospect theory, but because of changes in how the value of the result and uncertainty are focused depending on the situation. The decision is likely to be risk-averse under positive frame conditions because relative weight is placed on "certainty," rather than the "value of possible results," and risk-taking under negative frame conditions as a relative weight is placed on the "value of possible results," rather than on "reduction of uncertainty" (see Fig. 10.3).

The basic hypothesis of this contingent focus model is that the amount of attention increases when evaluating a result in negative aspects rather than in positive aspects. This hypothesis is consistent with the loss sensitivity principle that the decision-maker is more sensitive to a loss than a gain (Gärling and Romanus 1997, 1999).

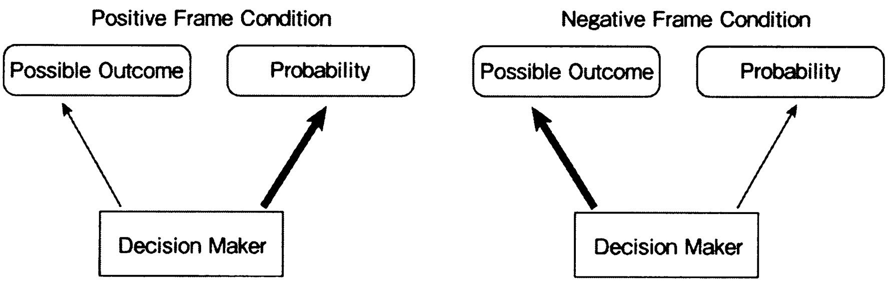


Fig. 10.3 Basic framework of contingent focus model. *Source*: Takemura (1994). Reproduced in part by author

4 Mathematical Expressions of Contingent Focus Model

First, the assumptions of the contingent focus model are described below. Assuming a set of alternatives is A and that its elements are mutually exclusive alternatives, a_1, a_2, a_3 …, Set A is described as $\{a_1, a_2, a_3 \ldots\}$. Subsequently, we consider the set of the results of adopting these alternatives: $X = \{x_1, x_2, x_3 \ldots\}$. The elements of X, for instance, include $x_1 = 200$ people will survive, $x_2 =$ no one will survive, and $x_3 = 600$ will people survive.

The result, x_i, of adopting the alternative, a_i, should depend at least on a state of some kind, $\Theta = \{\theta_1, \theta_2, \theta_3, \ldots\}$, which can be replaced with the probability distributions $p_1 = [p_{11}, p_{12}, p_{13}, \ldots]$, $p_2 = [p_{21}, p_{22}, p_{23}, \ldots]$, $p_3 = [p_{31}, p_{32}, p_{33}, \ldots]$, … on X if the probability distribution on Θ is known in decision making under risk (cf. Chap. 6). In problems of decision making under risk, the subject of selection is regarded as a Cartesian product, $X \times P$, assuming the set of the results of an alternatives, $X = \{x_1, x_2, x_3 \ldots\}$, and the set of probabilities of values in the interval [0,1].

At this stage, we consider only the cases in which result x_j occurs and does not occur for simplification, as in the studies of Tversky and Kahneman (1981), and the value of the situation in which result x_j occurs (i.e., the situation that no one will survive) as zero.

In the contingent focus model, then, when describing the value of adopting an alternative a_i, the description is possible only from the pair of a result and probability (x_j, p_j). In the example of the problem of an Asian disease of Tversky and Kahneman (1981), because the value of the situation in which no one will survive is zero, the description is made only from the situation (200 people, 1) in which 200 people will survive and the situation (600 people, 1/3) in which the result that 600 people will survive with a probability of 1/3 occurs.

The contingent focus model expresses the values under positive frame conditions as $U_{Po}[F_{Po}(x_j), G_{Po}(p_j)]$ and the values under negative frame conditions as $U_{Ne}[F_{Ne}(x_j), G_{Ne}(p_j)]$ (Takemura 1994). Subsequently, preference relations $\succsim_{Po}$ and $\succsim_{Ne}$, which satisfy the nature of weak order (preference relations satisfying comparability and transitivity), are assumed respectively as the preference relation under positive frame conditions and the preference relation under negative frame conditions. At this point, we assume that the value of one attribute is independent of the fixed value of the other attribute for all values of attributes under each set of frame conditions. In other words, for arbitrary $x_1, x_2 \in X$ and $p_1, p_2 \in$ P,

$$(x_1, p_1) \succsim_i (x_2, p_1) \Leftrightarrow (x_1, p_2) \succsim_i (x_2, p_2)$$

$$(x_1, p_1) \succsim_i (x_1, p_2) \Leftrightarrow (x_2, p_1) \succsim_i (x_2, p_2)$$

where $i = Po, Ne$.

Based on the assumptions presented thus far, the nature of weak order of the preference relation, $\succsim_i$, and the assumption that the equivalence class of $X \times P$ has order-dense countable subsets, the presence of functions F_i, G_i, and U_i that indicate the following relations defined on X, P, and $Re \times Re$ is derived. Apparently, these assumptions are necessary and sufficient conditions for the existence of the following function (Krantz et al. 1971). Consequently, for arbitrary $x_1, x_2 \in X$ and $p_1, p_2 \in P$,

$$(x_1, p_1) \succsim_i (x_2, p_2) \Leftrightarrow U_i[F_i(x_1), G_i(p_1)] \geq U_i[F_i(x_2), G_i(p_2)]$$

where U_i, $i = Po$, Ne are the monotone increasing functions of each argument. The equation above is a general form of the contingent focus model that explains the framing effect.

The contingent focus model considers the values under positive frame conditions $U_{Po}[F_{Po}(x_j), G_{Po}(p_j)]$. The values under negative frame conditions, $U_{Ne}[F_{Ne}(x_j), G_{Ne}(p_j)]$, are considered more specifically as the following functions.

$$U_{Po}\left[F_{Po}\left(x_j\right), G_{Po}\left(p_j\right)\right] = F_{Po}\left(x_j\right)^{\alpha Po} G_{Po}\left(p_j\right)^{\beta Po}$$

$$U_{Ne}\left[F_{Ne}\left(x_j\right), G_{Ne}\left(p_j\right)\right] = F_{Ne}\left(x_j\right)^{\alpha Ne} G_{Ne}\left(p_j\right)^{\beta Ne}$$

In the above, each of F_i, $i = Po$, Ne is a function to convert the value of results subjectively, G_i, $i = Po$, Ne are the functions to convert probability subjectively, and U_i, $i = Po$, Ne are the functions to evaluate F_i and G_i comprehensively.

Furthermore, in the contingent focus model, the preference relation between two alternatives is $F_i(x_j)^{\alpha i} G_i(p_j)^{\beta i}$, $i = Po$, Ne, which becomes equivalent to the size relation of the value of $F_i(x_j)^{wi} G_i(p_j)$ when assuming $w_i = \alpha_i/\beta_i$. In this case, α_i, β_i, and w_i are parameters that are inherent in each frame condition. In other words, $U_i[F_i(x_j), G_i(p_j)]$, $i = Po$, Ne is expressed with the functions, F and G, which are common in both framing conditions. In addition, exponents α_i, β_i, and w_i are inherent in each frame condition. Therefore, the framing effect is thought to occur because of the relation of how α_i and β_i are cast.

Accordingly, in the contingent focus model shown below holds true.

$$\begin{aligned}
(x_1, p_1) \succsim_i (x_2, p_2) &\Leftrightarrow F_i(x_1)^{\alpha i} G_i(p_1)^{\beta i} \geq F_i(x_2)^{\alpha i} G_i(p_2)^{\beta i}, i = Po, Ne \\
&\Leftrightarrow \alpha_i \log F_i(x_1) + \beta_i \log G_i(p_1) \geq \alpha_i \log F_i(x_2) + \beta_i \log G_i(p_2), i = Po, Ne \\
&\Leftrightarrow w_i \log F_i(x_1) + \log G_i(p_1) \geq w_i \log F_i(x_2) + \log G_i(p_2), i = Po, Ne \\
&\Leftrightarrow F_i(x_1)^{w_i} G_i(p_1) \geq F_i(x_2)^{w_i} G(p_2), i = Po, Ne
\end{aligned}$$

Therein, $w_i = \alpha_i/\beta_i$, $i = Po$, Ne, and $F(x_j)$ and $G(p_j)$ take positive values. The representation theorem that indicates the necessary and sufficient conditions of the contingent focus model in the above equations has been described by Takemura (1994).

5 Verification Experiment of the Contingent Focus Model

The most basic and important hypothesis among the hypotheses of the contingent focus model is the "focusing hypothesis" that the allocation of attention to the results and probabilities in decision-making influences the risk attitude. For verification of this hypothesis, Fujii and Takemura (2001a) conducted two experiments to determine whether the risk attitude of test subjects would change as predicted by the focusing hypothesis by experimentally manipulating the attention used for establishing recognition during the decision making process.

In Experiment 1, the test subjects consisted of 180 students and faculty members at Kyoto University. In all, six conditions, including two levels of frame conditions (positive and negative) and three levels of emphasis conditions (emphasis on results, no emphasis, and emphasis on risk), were adopted as experiment conditions. Then 30 test subjects were randomly assigned to each condition. As presented in Fig. 10.4, under the condition of emphasizing the result, letters indicating the result were enlarged and written in bold. They were then added with auxiliary words for additional emphasis. Similarly, under the condition of emphasizing the risk, the letters indicating probabilities were enlarged, written in bold, and added with auxiliary words to emphasize them.

Under the condition of emphasizing the result through such experiment manipulation, the amount of attention paid to the result is expected to increase in comparison to the risk-emphasizing condition. Therefore, based on the focusing hypothesis, the risk-taking tendency is predicted to be stronger under the result-emphasizing condition than the risk-emphasizing condition under both the positive and negative conditions. The results are presented in Table 10.1. Under the negative condition under the risk-emphasizing condition and no-emphasis condition, no clear tendency of risk aversion or acceptance was evident. This result is not necessarily consistent with the prediction of prospect theory. Meanwhile, the contingent focus model forecasted that a negative result would draw greater attention than a positive result. This result therefore supports the basic hypothesis of the contingent focus model rather than prospect theory. The results also reveal that the risk-taking tendency is stronger under the result-emphasizing condition than the risk-emphasizing condition under both the positive and negative conditions.

Outcome emphasis condition

☐ **Option A**: get **¥20,000** with certainty

☐ **Option B**: get **¥40,000** with probability 50%,
and ¥0 with probability 50%

Risk emphasis condition

☐ **Option A**: get ¥20,000 with **certainty**

☐ **Option B**: get ¥40,000 with **probability 50%,**
and ¥0 with **probability 50%**

Fig. 10.4 Manipulation of attentional focus and reflection effect. *Source*: Fujii and Takemura (2001a)

This result implies that the tendency to accept risk is stronger under the result-emphasizing condition than the risk-emphasizing condition, which is consistent with the prediction of the focusing hypothesis representing the basic hypothesis of the contingent focus model.

Table 10.1 Experimental result of the reflection problem

	Positive condition		Negative condition	
	Risk aversion	Risk seeking	Risk aversion	Risk seeking
	% (N)	% (N)	% (N)	% (N)
Risk emphasis condition	90.0 (27)	10.0 (3)	50.0 (15)	50.0 (15)
No emphasis condition	83.3 (25)	16.7 (5)	56.7 (17)	43.3 (13)
Outcome emphasis condition	63.3 (19)	36.7 (11)	30.0 (9)	70.0 (21)

Source: Fujii and Takemura (2001a)

In Experiment 2, the problem of the disease in Asia was used. The test subjects were 180 students and faculty members, and the experiment conditions, methods of presenting the problem, and other conditions were the same as those in Experiment 1. Table 10.2 presents the results. As in Experiment 1, the risk-taking tendency that is stronger under the result-emphasizing condition than under the risk-emphasizing condition irrespective of the frame conditions is apparent from the results. The above results prove to support the forecast of the contingent focus model.

Table 10.2 Experimental result for the Asian disease problem

	Positive condition		Negative condition	
	Risk aversion	Risk seeking	Risk aversion	Risk seeking
	% (N)	% (N)	% (N)	% (N)
Risk emphasis condition	70.0 (21)	30.0 (9)	40.0 (12)	60.0 (18)
No emphasis condition	60.0 (18)	40.0 (12)	56.7 (17)	43.3 (13)
Outcome emphasis condition	43.3 (13)	56.7 (17)	20.0 (6)	80.0 (24)

Source: Fujii and Takemura (2001a)

In addition to this, experiments in which the contingent focus model was examined include one in which the information was provided on personal computers (Takemura et al. 2001). In this experiment, the information was presented as shown in Fig. 10.5. The forecast using the contingent focus model was verified on the assumption of greater attention paid to letters presented with a higher frequency than others. Fujii and Takemura furthermore attempted to measure the focusing reaction directly using an eye movement measurement system and discovered selection results opposite of the prediction of prospect theory caused by changes in eye movements (Fujii and Takemura 2003). Additionally, Fujii and Takemura (2001) incorporated the concept of random utility in the contingent focus model and

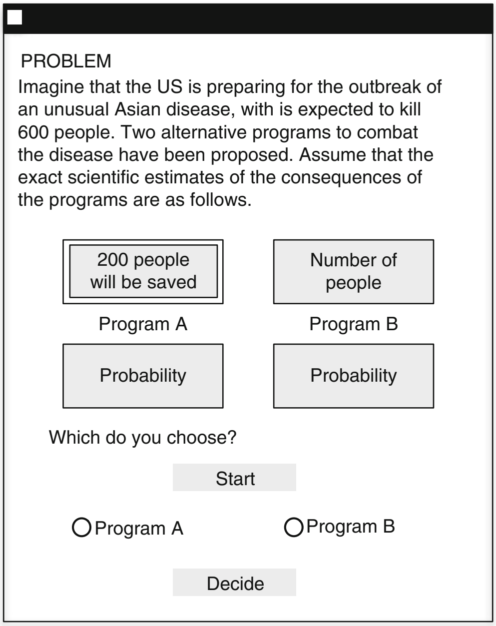


Fig. 10.5 Experimental task using information board computerized information board. *Source*: Takemura et al. (2001)

conducted meta-analysis based on some results of investigations and experiments in which the propriety of the contingent focus model was determined.

The contingent focus model demands further theoretical elaboration and empirical examination. This model presents policy implications that differ from those brought about by prospect theory. In fact, a study of social consensus-building (Fujii et al. 2002) and a study of consumer marketing policy (Takemura et al. 2004) were conducted based on the contingent focus model.

References

Fischhoff, B. (1983). Predicting frames. *Journal of Experimental Psychology: Learning, Memory, & Cognition, 9*, 103–116.

Fujii, S., & Takemura, K. (2001a). Risuku taido to chui : Jokyo izonteki shoten moderu ni yoru furemingu koka no keiryo bunseki [Risk attitude and attention: A psychometric analysis of framing effect by contingent focus model]. *Kodo Keiryogaku [The Japanese Journal of Behaviormetrics], 28*, 9–17.

Fujii, S., & Takemura, K. (2001b). Jokyo izonteki shoten moderu ni yoru furemingu koka no meta bunseki [Psychometric meta-analysis of framing effect by contingent focus model] Nihon kodo keiryo gakkai dai 29 kai taikai happyou ronbun syorokusyu [*Paper presented at the 29th International Meeting of the Psychometric Society (IMPS 2001)*] (pp. 164–167).

Fujii, S., & Takemura, K. (2003). Attention, frames condition and decision making under risk: An empirical test of the contingent focus model using an eye gaze recorder. *Paper presented at the Society for Judgment and Decision Making*, Vancouver, Canada.

Fujii, S., Takemura, K., & Kikkawa, T. (2002). Kimekata to goi keisei: Shakaiteki jiremma ni okeru rikoteki doki no yokusei ni mukete [Decision strategy and consensus formation: Towards inhibition of selfish motive in social dilemma]. *Dobokugakkai rombunshu [Proceedings of Japan Society of Civil Engineers], 709*, 13–26.

Gärling, T., & Romanus, J. (1997). Integration and segregation of prior outcomes in risky decisions. *Scandinavian Journal of Psychology, 38*, 289–296.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47*, 263–292.

Krantz, D. H., Luce, R. D., Suppes, P., & Tversky, A. (1971). *Foundations of measurement volume 1: Additive and polynomial representations*. New York: Academic.

Maule, A. J. (1989). Positive and negative decision frames: A verbal protocol analysis of the asian disease problem of Tversky and Kahneman. In H. Montgomery & O. Svenson (Eds.), *Process and structure in human decision making* (pp. 163–180). New York: Wiley.

Romanus, J., & Gärling, T. (1999). Do changes in decision weights account for effects of prior outcomes on risky decisions? *Acta Psychologica, 101*, 69–78.

Takemura, K. (1994). Furemingu Koka no Rironteki Setsumei: Risukuka Deno Ishikettei no Jokyo Izonteki Shoten Moderu [Theoretical explanation of the framing effect: Contingent focus model for decision-making under risk]. *Japanese Psychological Review, 37*, 270–293.

Takemura, K. (1998). Jokyo izonteki isikettei no teiseiteki moderu: Shinteki monosashi riron niyoru setsumei [Qualitative model of contingent decision-making: An explanation of using the mental ruler theory]. *Ninchi Kagaku [Cognitive Studies], 5*(4), 17–34.

Takemura, K. (2001). Contingent decision making in the social world. In C. M. Allwood & M. Selart (Eds.), *Decision making: Social and creative dimensions* (pp. 153–173). Dordrecht: Kluwer Academic.

Takemura, K., & Fujii, S. (1999). Contingent focus model of decision framing under risk. *Paper presented at 17th Biennial Conference on Subjective Probability, Utility and Decision Making*, Mannheim, Germany.

Takemura, K., Hu, K., & Fujii, S. (2001). Joho monitaringu ho niyoru jokyo izonteki syotenka moderu no kento [Examination of contingent focus model using a method of monitoring information acquisition]. *Paper presented at the Japan Society for Kansei Engineering*

Takemura, K., Wakayama, D., & Horiuchi, K. (2004). Kokoku juyo no suri shinnri moderu to de-ta kaiseki ho no kaihatsu: shohisha no handan to isikettei no shinri jikken to chosa kenkyu wo tsujite [Development of mathematical model and data analysis method of consumer advertising response: Through psychological experiment and survey research]. *Advertising Science, 45*, 153–172.

Tversky, A., & Kahneman, D. (1981). The framing of decisions and the psychology of choice. *Science, 211*, 453–458.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Uncertainty, 5*, 297–323.

Part VI
Decision-Making Process and Its Theory

Chapter 11
Decision-Making Process

This chapter presents a description of a decision-making process. The previous chapters have explained the decision-making phenomenon and theories to describe the phenomenon, which, however, have largely omitted a description of the process through which a decision is made. Accordingly, this chapter presents a description of how the psychological process used in decision making is explainable using a technique to follow the decision-making process. This chapter also elucidates the task variables and emotional factors affecting the decision-making process. The final section explains the changes made to cognition in psychological processes after decision making.

1 Multi-Attribute Decision-Making and Information Search

We assume a set of circumstances related to purchasing a product at a store. Presume, for instance, that we purchase a digital audio player. Consumers make a purchase decision after comparing multiple attributes such as the prices, number of recordable tracks, sound performance, and designs at stores or using catalogs. Such decision making after considering multiple attributes is called multi-attribute decision-making. Multi-attribute decision-making is presumably performed by obtaining information of multiple types.

Consumers' information search normally begins with internal information search to obtain related information from their memories. If adequate information is not stored in their memories, then external information is sought from sources such as information at stores and product catalogs (Engel et al. 1993; Mowen 1995). In this context, the word "internal" in internal information search refers to the brain structure that includes consumers' memories, and the word "external" in external information search refers to consumers' external environment. One report described in the literature has revealed that many consumers rely mostly on internal

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_11,

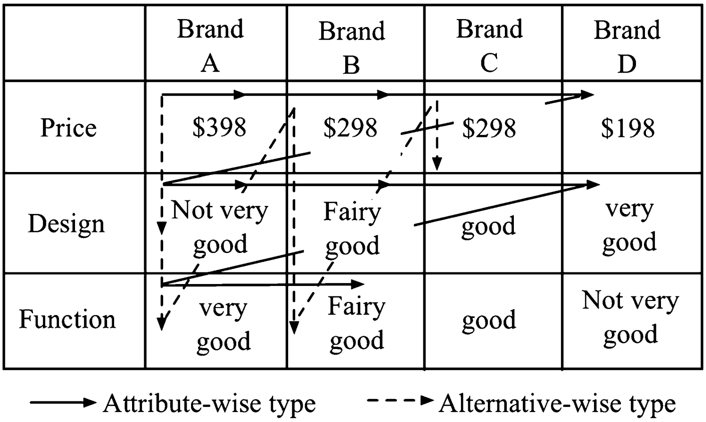


Fig. 11.1 Examples of attribute-wise information search and alternative-wise information search. *Source*: Takemura (1997a)

information that is sought from their memories when they make a decision related to auto-repair service. Only 40 % of them perform an external information search (Biehal 1983). Studies have also revealed that satisfaction with a previous purchase encourages total dependence of decision-making on internal information search (Engel et al. 1993), which is particularly the case in a repeated purchase of the same brand that was purchased previously.

Whether consumers make decisions solely based on internal information search is affected in part by their reliance on the knowledge they already have (Takemura 1997a). An example is the case of purchasing a PC for the first time. Consumers visiting stores have very little knowledge about products. Therefore, the information that is needed for decision making cannot be identified through internal search. Necessary information must therefore be obtained through external search using catalogs, asking store attendants, and using other means.

The methods of information search and the assessment of alternatives are closely related to each other. Decision making in the purchase of a television, for example, involves assessment of various televisions. Information about what attributes of a television are sought in what order strongly affects the overall assessment of the television. As presented in Fig. 11.1, the assessment of alternatives is expected to differ clearly when information search is performed for all attributes of all alternatives and when the search is limited to information related to a part of the attributes (Takemura 1997a). The assessment of alternatives and the results of decision making often vary between cases in which the most appropriate brand based on the most important attribute such as price is identified by acquiring information on, first, the prices of all brands, then on the second most important attribute (attribute-based information search) and cases in which the optimal brand is determined by searching for information related to each brand and then making a comprehensive evaluation (alternative-based information search) (Bettman 1979; Bettman et al. 1991; Takemura 1997a). The methods of information search and the assessment of alternatives are thereby inseparable.

2 Various Decision Strategies

An important concept in understanding the decision-making process from the perspective of information search is a decision strategy. A decision strategy determines the type of a series of mental operations used to assess and select alternatives. A decision strategy is also called decision heuristics. The idea of heuristics is in contrast to algorithms, which are action strategies that always result in optimal solutions.

In comparison to the use of algorithms, the use of heuristics often supports prompt and efficient problem solving. In some cases, however, inappropriate solutions or inconsistent and circumstantial decisions might result. Decision strategies for people's decision making are mostly heuristic, which are therefore often called decision heuristics. Although decision strategies are conceptually distinguished from information search strategies, they mutually correspond in most cases. In fact, studies of decision strategies frequently involve the analysis of information search patterns of decision makers, as described later (Klayman 1983; Bettman et al. 1991).

Preceding studies of decision-making process have shown that people's decision strategies rarely follow the procedures of utility maximization assumed by utility theory (Simon 1957; Abelson and Levi 1985; Gigerenzer and Selten 2001). Simon (1957) has stated that, rather than making decisions according to the principle of maximization or optimization for selecting the best from all available options, people opt for the principle of satisfaction to seek an alternative that satisfies them to a degree because of the limitation of their information processing capacity. Since then, several decision strategies derived primarily from the limitation of people's information processing capacity have been developed (Beach and Mitchell 1978; Payne 1976; Payne and Bettman 2004; Takemura 1985, 1996a, b, 1997a).

The decision strategies that have been developed to date include the following.

Additive strategy: A decision strategy of this type includes consideration of all alternatives at every level, which are assessed comprehensively to determine the best alternative. Additive strategies include a *weighted additive strategy* that places different weights on each attribute and an *equal weight strategy*, which is contrary.

Additive difference strategy: A decision strategy of this type compares the assessment values of each attribute between an arbitrary pair of alternatives. Three or more alternatives are paired and compared. Then the winners of the comparison are compared further in sequence in a tournament. The last remaining alternative is used.

Conjunctive strategy: A decision strategy of this type specifies necessary conditions of each attribute; any alternative failing to meet even one of its necessary conditions is excluded from the information processing and is rejected irrespective of the values of its other attributes. The first alternative that satisfies the necessary conditions of all of its attributes is the one to be selected based on this decision strategy.

Disjunctive strategy: A decision strategy of this type specifies sufficient conditions of each attribute; any alternative meeting at least one of its sufficient conditions is selected irrespective of the values of its other attributes.

Lexicographic strategy: A decision strategy of this type selects the alternative that is assessed as the highest in the most important attribute. Two or more alternatives scoring equally highest in the most important attribute are screened based on the next most important attribute. Alternatives with a slim margin to an extent, however, are regarded as ranking the same, and screening based on the next important attribute is called a *lexicographic semi-order strategy*.

Elimination by aspects (EBA) strategy: A decision strategy of this type examines whether each attribute satisfies its necessary conditions and rejects those failing to meet such conditions. This decision strategy is similar to the conjunctive strategy, except that it is an attribute-based strategy of examining multiple alternatives for each attribute.

3 Categories of Decision Strategies

Such various decision strategies have been identified, which are often categorized into two when studied: compensatory and non-compensatory strategies. A compensatory decision strategy makes a comprehensive evaluation, in which any attribute with a low assessment value is compensated by other attributes with a high value. Additive and additive difference strategies are included in this category. A compensatory strategy examines the information related to all alternatives. Non-compensatory decision strategy has no such compensatory relation among attributes, which includes conjunctive, disjunctive, lexicographic, and EBA strategies.

Decision results might vary depending on the examined alternatives and attributes based on the non-compensatory decision strategy, which might cause inconsistent decision making. Presuming, for instance, that a consumer makes a decision about television brands using the conjunctive strategy, then the conjunctive strategy selects the first alternative that has satisfied its necessary conditions. Therefore, the order of brands to examine is extremely important. Even if the consumer's most preferred television brand is available at another store, if one satisfying the necessary conditions is sold at the first store, the first will be purchased. Whether the consumer purchases the preferred brand the most is therefore likely to be affected by situation factors such as in-store product placement and store locations.

In actual decision making, compensatory and non-compensatory decision strategies tend to be combined according to the decision level. In other cases, consumers often narrow down the alternatives through elimination such as the EBA strategy to reduce their cognitive strain; they then use a compensatory strategy such as the additive method (Bettman 1979; Takemura 1996a, 1997a). As this process illustrates, decision strategies themselves might vary depending on the decision-making process. Such a type of decision making is sometimes called a multistage decision strategy (Takemura 1993).

4 How to Identify Decision Strategies

Methods to identify decision strategies include two: the verbal protocol method and the method of monitoring information acquisition (Bettman et al. 1991; Payne and Bettman 2004; Takemura 1997a).

The verbal protocol method is to have the test subject speak or write about the subject's own decision-making process, which is recorded and used to determine the type of decision strategy that was adopted. The verbal protocol method includes cases in which the subject speaks simultaneously during the decision-making process and cases in which memories are recorded immediately after decision making. A variation of this method is to present a list of decision strategies to the test subject in advance, who reports afterwards which strategies have been used.

A study of Takemura (1996a) that used the verbal protocol method, for example, had people who were considering the purchase of various products to keep a diary. A protocol suggesting that the conjunctive decision strategy was used at the final stage of making a decision was reported as shown in Table 11.1 in this study. The diary presented in Table 11.1 was written by a 22-year-old female college student who reported the process of purchasing a half-length coat in preparation for a trip to Canada.

Table 11.1 Example of verbal protocol

(omitted)
January 6, I heard on the radio that the temperatures in Canada and the East U.S. were below the freezing point and thought that I would purchase a very warm and waterproof coat
January 12, I went to a sporting goods store in Umeda. I went to the skiwear section and found half-length coats there. I found a half-length coat with a hood attached to it. The edge of the hood was decorated with faux fur, which is cute. The outer material is not waterproof, but is made water-resistant, so it should be okay even if it gets a little wet. I do not like the color very much, but it looks very warm. I decided to compromise on the color and purchased the half-length coat

Composed by author, based on Takemura (1996a)

Fig. 11.2 Eye-gaze equipment. *Source*: Okubo et al. (2006)

The method of monitoring information acquisition is to have the test subject freely search the information related to brands and analyze the attributes of the alternatives searched and the order of the search. This method includes the use of information displays, computer-based search systems, and measuring devices such as an eye camera (Bettman et al. 1991; Takemura 1996a, b, 1997a). Figures 11.2 and 11.3 depict photographs of the experiment of Okubo et al. (2006) on the method of monitoring information acquisition using an eye camera.

Analysis of data of an experiment using the method of monitoring information acquisition is performed by deducing, for instance, that examining the information related to all alternatives using the alternative-based information search will be an additive strategy or that reducing the alternatives to examine in sequence using the attribute-based information search will be an EBA strategy. As pointed out by Klayman (1983), some cases prevent the estimation of specific decision strategies based solely on information search. Combining the use of the verbal protocol method and the method of monitoring information acquisition in such a case allows identification to a considerable degree. At least whether the strategy is compensatory or non-compensatory or whether the information search is attribute-based or alternative-based is identifiable.

Fig. 11.3 Experiment using eye-gaze equipment. *Source*: Okubo et al. (2004)

5 Task Dependence of Decision Strategies

The results of preceding reports have indicated that these decision strategies are altered according to the properties of the task such as the number of alternatives and the number of attributes (Bettman 1979; Bettman et al. 1991; Payne and Bettman 2004; Takemura 1996a, b, 1997a). The reports suggest, in other words, that a small number of alternatives encourages the use of a complementary decision strategy and that a larger number of alternatives tends to promote the use of a non-complementary strategy. Takemura (1993), for example, conducted an experiment to assess decision making in the purchase of a radio-cassette recorder using the verbal protocol method. The numbers of alternatives and attributes were altered between two (4 and 10 for each) and the decision strategies used under each set of experimental conditions were compared. The decision strategies which were used under respective sets of conditions are presented in Table 11.2. Many test subjects adopted the style of combining multiple decision strategies at multiple stages. In general, non-compensatory strategies were used more often under the conditions of numerous alternatives or attributes. The analysis of these data also revealed that numerous alternatives would encourage the use of attribute-based decision strategies.

Table 11.2 Verbal protocols of decision strategies in each condition

	Four alternatives		Ten alternatives	
Participants	Four attributes	Ten attributes	Four attributes	Ten attributes
1	LEX, ADD	LEX, ADDc	EBA, ADDc	EBA, ADDc
2	CONJ	LEX-SEMI	CONJ, LEX-SEMI	LEX-SEMI
3	LEX, ADDc	LEX, CONJ	LEX-SEMI, ADDc	LEX-SEMI, ADDc
4	LEX, ADD	LEX, ADD	LEX	LEX, ADD
5	ADD	LEX-SEMI	LEX-SEMI	LEX-SEMI, ADD
6	ADD	CONJ	ADDc	CONJ, ADD, LEX
7	ADD	LEX	ADD	LEX, ADD
8	CONJ, ADD-DIF	ADDc	CONJ, ADD-DIF	CONJ, EBA, ADD-DIF
9	LEX-SEMI, ADD	LEX-SEMI	LEX-SEMI	LEX-SEMI
10	ADD	CONJ, ADD	LEX	EBA
11	ADD	LEX-SEMI	ADD, LEX	LEX-SEMI, ADD
12	LEX, ADD, CONJ	ADD, CONJ	ADD	ADD, LEX-SEMI, CONJ
13	ADD	ADD, DISJ	LEX, ADD	CONJ, DISJ
14	LEX, ADD	ADDc	LEX, LEX-SEMI	LEX, LEX-SEMI
15	ADD	LEX, ADDc	LEX-SEMI, LEX	LEX-SEMI, ADDc
16	CONJ, ADD	LEX	ADD	CONJ
17	LEX, EBA, ADD	CONJ, LEX	LEX	LEX, LEX-SEMI, EBA, ADD
18	CONJ, LEX, ADD-DIF	CONJ, ADD	CONJ, LEX	CONJ, ADD
19	LEX, ADDc	ADDc	LEX	ADDc
20	ADDc	EBA, DISJ	ADD, EBA, LEX	EBA, ADDc
21	ADD	EBA	EBA	LEX, EBA, CONJ
22	ADDc	CONJ, ADDc	LEX-SEMI	LEX-SEMI, ADD-DIF, CONJ
23	ADD	CONJ	LEX, CONJ	LEX, LEX-SEMI, EBA

Notes: *ADD* additive strategy, *ADDc* additive strategy based on the salient attributes, *ADD-DIF* additive-difference strategy, *CONJ* conjunctive strategy, *DISJ* disjunctive strategy, *LEX* lexicographic strategy, *LEX-SEMI* lexicographic-semiorder strategy, *EBA* elimination-by-aspects strategy
Source: Takemura (1993)

The reason for such alternation of decision strategies based on changes in the number of alternatives or attributes is interpreted as an effort to avoid the cognitive strain from information overload caused by processing a large amount of information required by the conditions of many alternatives or attributes by selecting simpler decision strategies with less burden of information processing.

The state of information overload is thereby presumed to simplify the decision strategies selected, leading to varying results of decision making (Information overload not only simplifies decision strategies, but tends to provoke the avoidance of the state of decision making. Takemura (1996a) conducted an in-store interview survey of consumers at supermarkets and reported that those consumers in confusion in the state of information overload tended to leave the store to avoid conflicts).

Not only the number of attributes and alternatives, but factors such as the form of information display in a decision-making task, operation of decision-making task variables such as reaction modes of decision making, and operation of motivation variables such as the level of psychological involvement of the decision maker in the decision making have been found to affect decision strategies (Abelson and Levi 1985; Bettman et al. 1991; Engel et al. 1993; Takemura 1996a, b).

6 Emotions and Decision Strategies

Henry Montgomery

Born in 1943. Graduated and earned his MA in 1971 and his Ph.D. in 1975 from the Department of Psychology, Stockholm University. After serving at Gothenburg University from 1971 to 1991, he was working as a professor at the Department of Psychology, Stockholm University. He made important contributions to decision process theory. He started the process-tracing study of decision making in the early 1970s with Ola Svenson and Tommy Gärling. In 1983, he proposed the dominance search model that previously described. His research deals with mental processes and structures associated with people’s values, judgments and decisions, with applications in economic, medical, organizational and political contexts.

Photograph given by Professor Emeritus Henry Montgomery

Decision strategies are evidently influenced also by the emotions of the decision maker (Cohen and Areni 1991; Luce et al. 2001; Payne and Bettman 2004; Takemura 1996c, 1997b).

Isen and Means (1983) studied the effects of positive emotions (good feelings) on decision strategies. The study revealed that the test subjects who had received false feedback that they had been successful in a sensorimotor task (positive emotion group) would take a longer time for fictitious selection of an automobile and search less for information about the decision than those test subjects who had not received feedback (control group).

This study also revealed the use of the verbal protocol method by which the test subjects with positive emotions, as opposed to those in the control group, tended to make decisions through an EBA strategy of sequentially eliminating the alternatives based on the attributes that were to be emphasized. Despite the lack of certainty in generating optimal decisions, EBA decision strategies involve less of a cognitive burden, which was presumably the reason for positive emotions to encourage the use of strategies of such a nature.

Additionally, Takemura (1987) showed that those test subjects voluntarily reported in a questionnaire on their feelings that they were feeling good on the day took less time to select their fictitious date partners, searched less for information, and were more confident after making their decision than the counterpart group. Forgas (1991) made a similar study of the task of opposite-sex partner selection and also reported that positive emotions would reduce the time of decision making and promote the use of simple decision strategies in selection.

Takemura (1988) further demonstrated that the test subjects given false feedback that they had scored highly (the standard score of 67.5 and ranked within top 4 %) on a mental test (actually, Uchida–Kraepelin psychodiagnostic test) took a shorter time to make decisions in both actual selection (high involvement condition) and hypothetical selection (low involvement condition) of a radio-cassette recorder, searched less for information, and re-examined less information than those in the control group (Fig. 11.4 presents the mean number of information searches). The result of this experiment added with the involvement factor remained the same as a result of the study of Isen and Means in the effects of positive emotions.

In addition to positive emotions, Takemura (1988) considered the effects of negative emotions. This study operated negative emotions by giving the subjects false feedback that they had scored low (the standard score of 32.5 and ranked in the lowest 4 %) in the mental test. The result confirmed the interaction between negative emotions and the involvement factor. In other words, the test subjects with negative emotions were found to take a longer time to make decisions, search for more information, and re-examine more information under the high involvement condition, and conversely, take a shorter time in decision making, search less for information, and re-examine less information under the low involvement condition in comparison to the control condition group.

This method operates emotions by feeding back a test result, which, in a sense, is a social stimulus and might have affected cognitive variables other than emotions such as self-image and self-esteem. Accordingly, Akiyama and Takemura (1994)

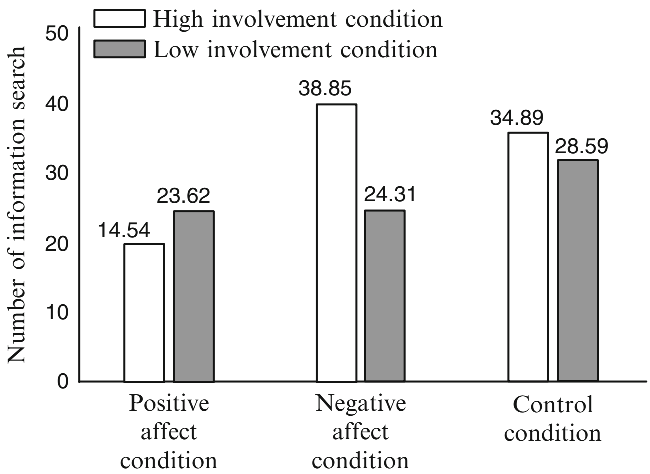


Fig. 11.4 Mean number of information searches in each condition. *Source*: Takemura (1988)

conducted a similar experiment using garbage odors to solve this problem. The result was fundamentally the same related to the interaction between negative emotions and involvement.

Recent studies have begun to examine other dimensions such as the dimension of arousal rather than the dimension of positive and negative emotions alone; the effect reportedly varies depending on the level of arousal (Lewinsohn and Mano 1993). As noted earlier, the effect of emotions has been found to interact with the effect of involvement (Takemura 1988). Evidently, emotions therefore affect decision strategies, and how they affect them is thought to change according to the degree of involvement or arousal.

7 Justification of Decisions and Process After Decision Making

Ola Svenson

Born in 1939. Graduated and earned his MA in 1967 and his Ph.D. in 1971 from the Department of Psychology, Stockholm University. After serving at Lund University from 1987 to 1993, he was working as a professor at the Department of Psychology, Stockholm University. He is the head of the Risk Analysis, Social and Decision Research Unit, Stock Holm University.

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He is also a researcher at Decision Research, Oregon USA. His main research interests cover basic and applied research in human decision processes, risk- and accident analysis and social psychology. He made important contributions to decision process research.

Photograph taken by author

The process leading up to decision making has been described so far with specific examination of the concept of decision strategies. The following describes the psychological changes that occur after decision making.

Festinger (1957) proposed a theory called cognitive dissonance theory and described the qualitative characteristics of the psychological changes after decision making. He used the term "cognition" to refer to all knowledge collectively, including opinions, beliefs, and emotions about oneself in a personal environment, and hypothesized that people would attempt to eliminate dissonance when it arose between units of cognition. He argued that the state of dissonance was uncomfortable, and that effort to reduce this discomfort would cause changes in cognition, variation of behavior, addition of new cognition, and selective exposure to new information.

Festinger believed that dissonance was likely to arise after decision making when action had already been taken in many cases, thereby prompting measures to eliminate the dissonance. For this reason, the appeal of an alternative after selection is expected to increase from the state before the selection. A study of cognitive dissonance that asked people who had just bought and those who were about to buy their betting tickets at a racetrack about the probability of the winning of their bets indicated that they believed in their chance of winning more strongly after the purchase of their tickets than they did before the purchase (Knox and Inkster 1968). This result can also be interpreted as the justification of one's decision to eliminate dissonance that arose after the decision was made.

Festinger and Carlsmith (1959) reported that people who were paid $1 for telling other experiment participants an untruth that a task that was actually dull was interesting tended more to justify their action and believed that the task was interesting than those who were paid $20 for the same action.

This result is consistent with cognitive dissonance theory. The reason is that telling that a dull task is interesting despite the low pay-off of $1 is considered more dissonant than receiving $20 for the same action, making the subject susceptible to a change of cognition.

Cognitive dissonance theory covers a wide area of decision making studies, and research using the concept of cognitive dissonance continues to date (Harn-Jones and Mills 1999; Matz and Wood 2005). While cognitive dissonance theory specifically examines the process of justification after decision making, some researchers including Montgomery (1983, 1993), Svenson (2006), and Luce et al. (2001) have pointed out the effect that justification of decision making has on decision strategies used before decision making.

References

Abelson, R. P., & Levi, A. (1985). Decision making and decision theory. In G. Lindzey & E. Aronson (Eds.), *The handbook of social psychology* (3rd ed., Vol. 1, pp. 231–309). New York: Random House.

Akiyama, M., & Takemura, K. (1994). Hukai kanjo to kanyo ga ishikettei katei ni oyobosu eikyo [Effect of negative affect and involvement on the decision making process]. *Japanese Journal of Experimental Social Psychology, 34*, 58–68.

Beach, L. R., & Mitchell, T. R. (1978). A contingency model for the selection of decision strategies. *Academy of Management Review, 3*, 439–449.

Bettman, J. (1979). *An information processing theory of consumer choice*. Reading: Addison Wesley.

Bettman, J., Johnson, E. J., & Payne, J. W. (1991). Consumer decision making. In T. S. Robertson & H. H. Kassarjian (Eds.), *Handbook of consumer behavior* (pp. 50–79). Englewood Cliffs: Prentice Hall.

Biehal, G. J. (1983). Consumers' prior experiences and perceptions in auto repair choice. *Journal of Marketing, 47*, 82–91.

Cohen, J. B., & Areni, C. S. (1991). Affect and consumer behavior. In T. S. Robertson & H. H. Kassarjian (Eds.), *Handbook of consumer behavior* (pp. 188–240). Englewood Cliffs: Prentice Hall.

Engel, J. F., Blackwell, R. D., & Miniard, P. W. (1993). *Consumer behavior* (7th ed.). New York: Dryden.

Festinger, L. (1957). *A theory of cognitive dissonance*. Stanford: Stanford University Press. [Suenaga, T. (Trans. supervisor) (1965). *Ninchiteki Fukyowa No Riron: Shakai Shinrigaku Josetsu*. Seishin Shobo].

Festinger, L., & Carlsmith, J. M. (1959). Cognitive consequences of forced compliance. *Journal of Abnormal and Social Psychology, 58*, 203–210.

Forgas, J. P. (1991). Affective influence on partner choice: Role of mood in social decisions. *Journal of Personality and Social Psychology, 61*, 708–720.

Gigerenzer, G., & Selten, R. (Eds.). (2001). *Bounded rationality: The adaptive toolbox*. Cambridge: MIT.

Harn-Jones, E., & Mills, J. (Eds.). (1999). *Cognitive dissonance: Progress on a pivotal theory in social psychology*. Washington, DC: American Psychological Association.

Isen, A. M., & Means, B. (1983). The influence of positive affect on decision making strategy. *Social Cognition, 2*, 18–31.

Klayman, J. (1983). Analysis of predecisional information search patterns. In P. Humphreys, O. Svenson, & A. Vari (Eds.), *Analyzing and aiding decision processes* (pp. 401–414). Amsterdam: North-Holland.

Knox, R. E., & Inkster, J. A. (1968). Postdecision dissonance at post time. *Journal of Personality and Social Psychology, 8*, 319–323.

Lewinsohn, S., & Mano, H. (1993). Multi-attribute choice and affect: The influence of naturally occurring and manipulated moods on choice processes. *Journal of Behavioral Decision Making, 6*, 33–51.

Luce, M. F., Bettman, J. R., & Payne, J. W. (2001). *Emotional decisions: Tradeoff difficulty and coping in consumer choice: Monographs of journal of consumer research 1*. Chicago: University of Chicago Press.

Matz, D., & Wood, W. (2005). Cognitive dissonance in groups: The consequences of disagreement. *Journal of Personality and Social Psychology, 88*, 22–37.

Montgomery, H. (1983). Decision rules and the search for a dominance structure: Towards a process model of decision making. In P. C. Humphreys, O. Svenson, & A. Vari (Eds.), *Analyzing and aiding decision processes* (pp. 343–369). Amsterdam: North-Holland.

Montgomery, H. (1993). The search for a dominance structure in decision making: Examining the evidence. In G. A. Klein, J. Orasanu, R. Calderwood, & C. E. Zsambok (Eds.), *Decision making in action: Models and methods* (pp. 182–187). Norwood: Ablex.

Mowen, J. C. (1995). *Consumer behavior* (4th ed.). Englewood Cliffs: Prentice Hall.

Okubo, S., Morogami, S., Takemura, K., & Fujii, S. (2006). Kanjo Ga Joho Kensaku Ni Ataeru Eikyo No Jikken Kenkyu: Aikamera Ni Yoru Katei Tsuiseki Wo Mochiite [An experimental study of the effects of emotions on information search: Using eye camera-based process tracking]. *Collection of Lecture Papers for 2004 Entaateimento Kansei Waakushoppu* [*Entertainment Sensitivity Workshop*] (pp. 14–19).

Payne, J. W. (1976). Task complexity and contingent processing in decision making: An information search and protocol analysis. *Organizational Behavior and Human Performance, 16*, 366–387.

Payne, J. W., & Bettman, J. R. (2004). Walking with the scarecrow: The information-processing approach to decision research. In D. J. Koehler & N. Harvey (Eds.), *Blackwell handbook of judgment and decision* (pp. 110–132). Malden: Blackwell Publishing.

Simon, H. A. (1957). *Administrative behavior: A study of decision making process in administrative organization*, New York, NY: McMillan.

Svenson, O. (2006). Pre-and postdecision construction of preferences: Differentiation and consolidation. In S. Lichtenstein & P. Slovie (Eds.), *The construction of preference* (pp. 356–371). New York: Cambridge University Press.

Takemura, K. (1985). Ishikettei sutorateji jikko ni okeru meta ninchi katei moderu [A metacognitive model for the implementation of decision strategies]. *Doshisha shinri [Doshisha Psychological Review], 32*, 16–22.

Takemura, K. (1987). Ishikettei Katei No Kenkyu (X): Kanjo To IshiKettei Katei No Kanrensei [Study of decision-making process (X): relationship between emotions and decision-making process]. *Collection of Papers Presented at the Fourth Annual Meeting of the Japanese Cognitive Science Society* (pp. 56–57).

Takemura, K. (1988). Ishikettei Katei No Kenkyu (XII): Kanjo Oyobi Kanyo Ga IshiKettei Katei Ni Okeru Joho Kensaku Kodo Ni Oyobosu Koka [Study of decision-making process (XII): The effects of emotions and involvement on the information search behavior in decision-making process]. *Collection of Papers Presented at the 36th Annual Meeting of the Japanese Group Dynamics Association* (pp. 23–24).

Takemura, K. (1993). Protocol analysis of multistage decision strategies. *Perceptual and Motor Skills, 77*, 459–469.

Takemura, K. (1996a). *Ishikettei no shinri: sono katei no kenkyu [Psychology of decision-making: Investigation of its process]*. Tokyo: Fukumura Shuppan.

Takemura, K. (1996b). Ishikettei to sono sien [Decision making and decision support]. In S. Ichikawa (Ed.), *Ninchi shinrigaku 4kan shiko [Cognitive Psychology 4: Thought]* (pp. 81–105). Tokyo: University of Tokyo Press.

Takemura, K. (1996c). Pojiteibu na kanjo to shakai kodo [Positive affect and social behavior]. In S. Tsuchida & K. Takemura (Eds.), *Shiko to kodo, ninchi, seiri: Kanjo no shakai shinrigaku [Affect, cognition, behavior, and physiology]* (pp. 151–177). Tokyo: Seishin Shobo.

Takemura, K. (1997a). Shohisha no joho tansaku to sentakushi hyoka [Alternative evaluation and consumer buying decision]. In T. Sugimoto (Ed.), *Shohisha rikai no tame no shinrigaku [Understanding in consumer psychology]* (pp. 56–72). Tokyo: Fukumura Shuppan.

Takemura, K. (1997b). Shiko, handan to kanjo [Affect, judgment, and thought]. In H. Kaiho (Ed.), *Atatakai ninchi no shinrigaku: ninchi to kanjo no yusetsu gensyo no hushigi [Psychology of hot cognition]* (pp. 77–97). Tokyo: Kaneko Shobo.

Chapter 12
Behavioral Decision Theories that Explain Decision-Making Processes

This article will explain various behavioral decision theories related to decision-making processes. As explained last time, decision strategies for decision-making take numerous forms. The selection of decision strategies is affected by such condition factors as the number of alternatives and the number of attributes. Numerous models have been proposed to explain the psychological processes related to such a selection of decision strategies. This chapter will introduce some models that are useful to explain decision-making processes. It ends with some speculation about the future of modern behavioral decision theories while referring to their relation with fields related to neuroscience, such as neuroeconomics, that have been developed in recent years.

1 Dominance Structure Search Model

The dominance structure search model proposed by Montgomery (1983, 1993) explains the mechanism of selecting a range of decision strategies. Montgomery argued that being able to justify a decision that one had made was an important goal of decision making and that various strategies could be used to justify the decision that was made. This model used such a decision strategy that would convince the decision-maker about the reasons for opting for the alternative that he/she had selected over other alternatives. We examine the case of making the decision to purchase a personal computer (PC) as an example. The dominance structure search model includes the assumption that when we compare and consider different kinds of PCs that we might purchase, the decision is made by selecting such a decision strategy that convinces us that the PC we intend to purchase eventually is superior to others.

In fact, the decision-maker has been able to determine the kind of situation in which each of the various decision strategies would be appropriate. Adelbratt and Montgomery (1980) presented multiple decision strategies to decision-makers and had them assess the degree of propriety of a particular strategy under

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_12,

particular circumstances. The study found consistency between the observation of behavior as to which strategies were adopted by the test subjects and the assessment of the possibility of adopting such strategies.

According to Montgomery (1983), the goal of decision-making is to search for dominant alternatives. Dominance corresponds to an order relation between the subjects in terms of a certain attribute. When a decision-making problem is a multi-attribute decision-making problem that can be expressed with multiple attributes, if one of the alternatives is desirable at least to an equivalent degree in terms of at least one attribute, then the former is said to be dominant over the latter. If, for instance, one PC is more desirable than another in the aspects of price, performance, and design, then it is "dominant" over the other one. The dominance structure search model adopts a dominant alternative if available; otherwise, it assumes that a dominant alternative is created by the decision-maker through psychological reorganization of decision-making problems and the use of various decision strategies.

To facilitate the understanding of Montgomery's dominance structure search model, Kobashi (1988) rearranged the flowchart of this model into the one presented in Fig. 12.1 and summarized the stages of the model as shown below.

I. *Preliminary editing*: Specify the alternatives and attribute groups in a decision-making problem. In preliminary editing, the decision-maker evaluates the importance of attributes and might eliminate less-important attributes. Alternatively, conjunctive (CON) and elimination by aspects (EBA) decision strategies are used to eliminate less-promising alternatives at the preliminary editing stage.
II. *Selection of promising alternatives*: Select one alternative that is likely to satisfy the dominance from the alternatives that have gone through the preliminary editing phase. Because the alternatives whose attributes are particularly desirable are more likely to be considered promising, at this stage, disjunctive (DIS), lexicographic (LEX), and EBA decision strategies are likely to be used.
III. *Dominance test*: At this stage, promising alternatives are tested for their dominance over all other alternatives. In other words, this stage uses the dominance rule (DOM) to select dominant alternatives. Failure on this test results in the exercise of dominance structuring of the next stage, IV.
IV. *Dominance structuring*: This process is implemented following failure on a dominance test as a subroutine at the dominance test stage. The goal is to eliminate the disadvantages of promising alternatives, i.e., factors preventing them from becoming dominant alternatives. An alternative is canceled if it fails in the process of dominance structuring.

In the final stage of the dominance structuring process, a dominance structure is sought through operations of dominance structuring. A dominance structure is a state in which alternatives are given dominance through psychological compensation, although no single likely alternative is dominant over all other alternatives. The four operations included are "de-emphasizing," "bolstering," "cancellation," and "collapsing" (Kobashi 1988).

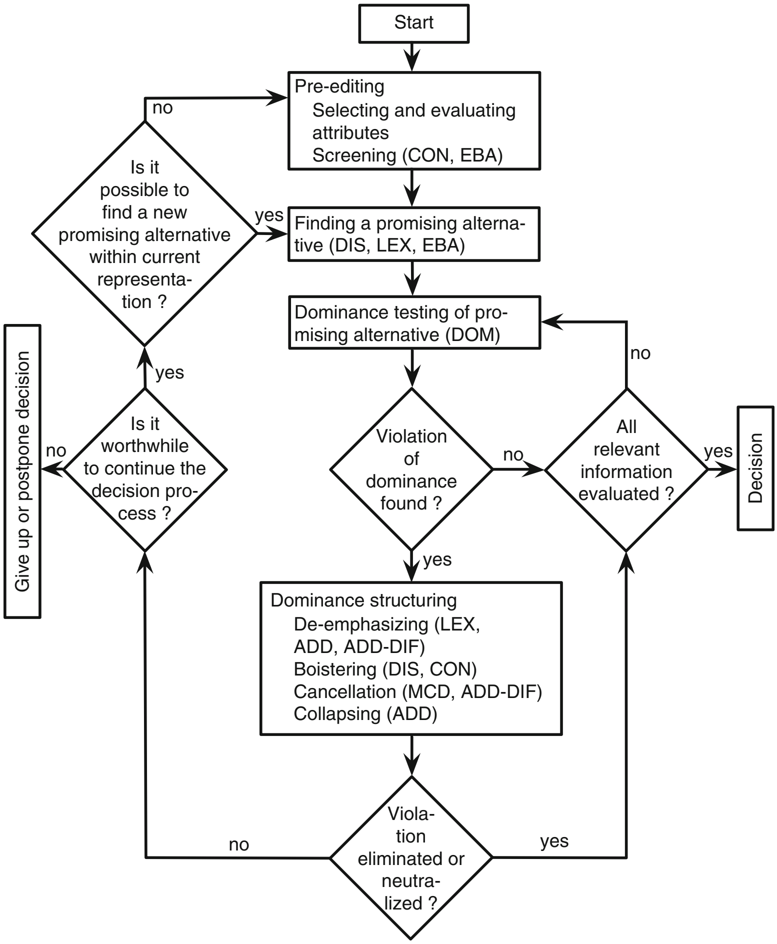

Fig. 12.1 Dominance search model of decision making. *Source*: Montgomery (1983)

"De-emphasizing" is to obscure a disadvantage of a promising alternative through, for instance, cognitive processing to consider the attribute constituting the disadvantage insignificant or that the disadvantage might not actually exist or appear. Lexicographic (LEX), additive utility (AU), and addition of utility difference (AUD) strategies correspond to this operation. An example of this is purchasing a PC of which only the design is not satisfactory. The design attributes are ignored when using the lexicographic decision strategy based on such attributes as price and functionality.

"Bolstering," is the opposite of "de-emphasizing," which is a cognitive operation that emphasizes the advantages of an alternative and disadvantages of other alternatives. Disjunctive (DIS) and conjunctive (CON) strategies correspond to this operation. When purchasing a PC, for example, the decision might be made using the disjunctive strategy based only on the attributes of notable CPU functions or using the conjunctive strategy to select one alternative that satisfies the minimum criteria for the price, functionality, and design.

"Cancellation" is an operation to offset a disadvantage of a particular attribute by an advantage of another attribute to seek a dominant structure. The decision strategies of the addition of utility difference (AUD) and winning percentage maximization (MNA: maximizing number of attributes with a greater attractiveness) correspond to this operation. Winning percentage maximization is to select an alternative that is left after a tournament among those alternatives with numerous good attributes selected through paired comparison. In the case of a low-priced and highly functional PC and a high-priced and less functional PC, for instance, the advantages and disadvantages of these two attributes are offset in winning percentage maximization. Taking this into account, if one has better design, then it is dominant over the other, thereby forming a dominant structure.

"Collapsing" is to reduce the desirability of two or more attributes and integrate them at a level that is easier to understand. The additive utility (AU) decision strategy corresponds to this operation. In the example of purchasing a PC, the AU strategy might be used to seek dominant alternatives by expressing them in respective acceptable prices.

As described so far, Montgomery's dominance structure search model includes the assumption that the decision-maker seeks a dominant alternative. If no dominant alternative is found, then the decision-maker develops a state in which the disadvantage of a likely alternative is compensated using a decision strategy operation. In this sense, this model is regarded as emphasizing the aspect of motivation based on "acceptance" and "justification" rather than the perspective of human data processing.

2 Adaptive Decision-Maker Model

Payne et al. (1993) has proposed the adaptive decision-maker model. Rather than emphasizing the process of motivation based on justification just as the dominance structure search model, this model specifically examines the aspect of data processing efficiency in decision-making. This model includes the assumption that, to adapt to the situation, a decision-maker selects an appropriate decision strategy by considering the cost (expenditure) such as cognitive load and the benefit that an accurate selection will be made through the decision strategy.

The original idea for this model dates back to the contingent model for decision strategy selection of Beach and Mitchell (1978). Payne et al. extended and elaborated the basic idea of this model to allow computer simulation. Payne et al. reported that a particular decision strategy will be selected under certain conditions as a result of a decision-maker trading off the degree of cognitive effort required for the decision and the propriety (accuracy) of the decision-making. In other words, this model includes the assumption that a decision-maker places more weight on either the accuracy or ease (reduced cognitive effort) depending on the circumstances when selecting a decision strategy.

Figure 12.2 presents the case of the tradeoff between accuracy and effort described by Payne et al. The horizontal axis of this figure represents the number of operations in elementary information processes (EIP) such as comparison and removal, which constitutes the indicator of cognitive effort (cognitive load). Payne et al. have stated that EIP involves the operations presented in Table 12.1. The vertical axis of Fig. 12.3 is the indicator of the relative accuracy of the results of decision. The indicator of relative accuracy is the value resulting from dividing the difference between the expected values of random selection and the decision strategy by the difference between the expected values of random selection and the expectation maximization strategy. More specifically, the relative accuracy is defined operationally by the indicator that takes the value of 1 when the result is identical to the load addition strategy and 0 when the response is entirely random.

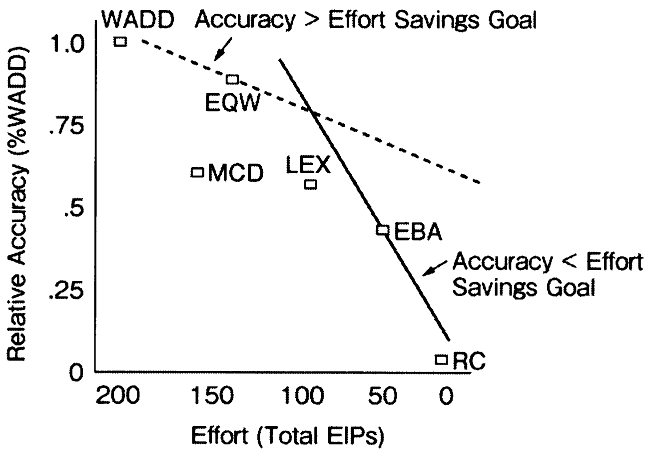

Fig. 12.2 Selection of strategies with different goals for effort savings and accuracy. *Notes*: *WADD* Weighted additive rule, *EQW* Equal weight rule, *LEX* Lexicographic rule, *EBA* Elimination by-asrects rule, *MCD* Majority of confirming dimensions rule, *RC* Random rule. *Source*: Payne et al. (1993)

Table 12.1 Elementary EIPS used in decision strategies

Read	Read an alternative's value on an attribute into STM
Compare	Compare two alternatives on an attribute
Difference	Calculate the size of the difference of two alternatives for an attribute
Add	Add the values of an attribute in STM
Product	Weight one value by another (multiply)
Eliminate	Remove an alternative or attribute from consideration
Move	Go to next element of external environment
Choose	Announce preferred alternative and stop process

Source: Payne et al. (1993)

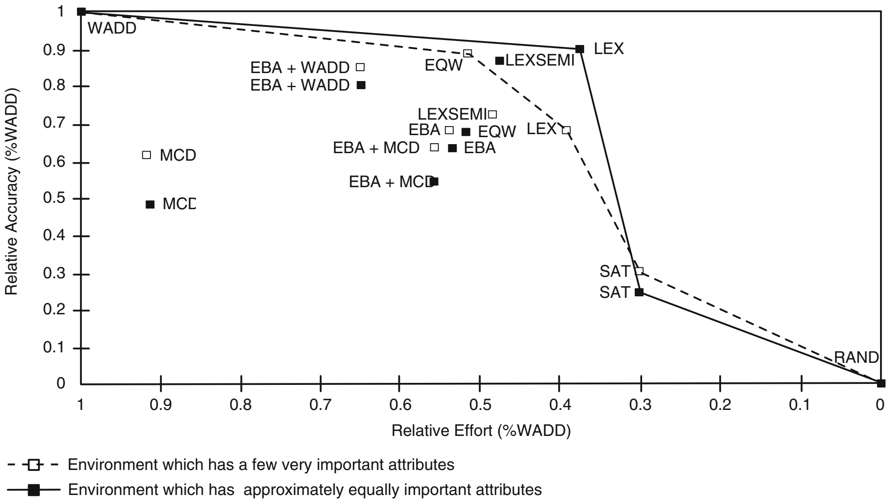

Fig. 12.3 Effort and accuracy levels for various strategies for different decision environments. *Notes*: *WADD* Weighted additive rule, *EQW* Equal weight rule, *LEX* Lexicographic rule, *LEXSEMI* Lexicographic-semiorder rule, *CON* Conjunctive rule, *MCD* Majority of confirming dimensions rule, *EBA* Elimination by-asrects rule, *RC* Random rule. *Source*: Payne et al. (1993)

The dotted line in Fig. 12.2 shows the indifference curve (assumed as a straight line in this case) when accuracy is focused in the relative weight of accuracy and reduced cognitive effort. The solid line in Fig. 12.2 is the indifferent straight line when the relative weight is placed on the reduced cognitive effort. Although each indicates dominance on the upper right side, the steeper the slope of the indifferent straight line, the more focused the reduced cognitive effort. This figure suggests that the emphasis on accuracy tends to result in the selection of equal load addition strategy. The emphasis on reduced cognitive effort is likely to engender the adoption of an EBA strategy.

Payne et al. (1993) performed a computer simulation using the Monte Carlo method by varying the numbers of alternatives and attributes and the degrees of dispersion of decision-making loads. The cognitive effort (operationally defined by the number of EIP operations) for the exercise of each strategy and relative accuracy (operationally defined by the indicator that takes the value 1 when the result is identical to the load addition strategy and 0 when the response is entirely random) of the results of decision were included in the conditions for the calculation.

Figure 12.3 depicts the tradeoff between cognitive effort and accuracy on the condition of large and on the condition of small dispersion of load on the attributes of decision-making. The vertical axis is the relative accuracy when the accuracy of load addition strategy is 1. The horizontal axis is the relative effort when the number of operations in the load addition strategy is 1. The two decision strategies added with a plus sign in Fig. 12.3 represent a multistage decision strategy by which a strategy is selected after narrowing the alternatives down to three or fewer with the first strategy.

According to Fig. 12.3, if accuracy is emphasized, then the lexicographic strategy is dominant over other decision strategies when the dispersion of load on the attributes is large, and the equal load addition model is dominant over other decision strategies when the load dispersion is small. It is also suggested that more cognitive effort is required when the dispersion of load on the attributes is large rather than when it is small. Furthermore, although the relative accuracy of decision is variable because of the load dispersion, the cognitive effort does not change significantly.

Figure 12.4 presents the results of relative accuracy and cognitive effort through various strategies when the quantities of attributes and alternatives have been changed. As indicated in Fig. 12.4, although the load addition strategy allows accurate decisions, an increase in the numbers of alternatives and attributes requires an extremely large amount of cognitive effort. In the lexicographic strategy, an increase in the numbers of alternatives and attributes only slightly requires any cognitive effort, and simultaneously, maintains the accuracy to a certain degree. Furthermore, non-compensatory decision strategies such as lexicographic and conjunctive approaches would not require as much cognitive effort as the load addition strategy would, even with an increase in the numbers of alternatives and attributes.

Experimental studies in the past demonstrated that an increase in the numbers of alternatives and attributes would raise the percentage of non-compensatory decision strategies being adopted. This phenomenon can be interpreted consistently based on the results of simulation. In other words, small numbers of alternatives and attributes would not require much cognitive effort, for which compensatory strategies with high accuracy such as load addition tend to be employed. For large numbers of alternatives and attributes, compensatory decision strategies requiring an extremely large amount of cognitive effort would not be adopted; rather, non-compensatory strategies requiring less cognitive effort would likely be used.

Payne et al. (1993) and Payne and Bettman (2004) associated the results of this simulation with the results of a number of psychological experiments, traded off the accuracy of selection and cognitive effort, and concluded that the decision-makers were adaptively selecting decision strategies. This model of Payne et al. alone, however, does not necessarily explain the situation-dependent assessment of alternatives and decision-making. The framing effect and emotional and motivational effects have not been fully explained to date. Yet, the model of Payne et al. allows quantitative forecasting of what kinds of decision strategies would be adopted in what situations and what kinds of decisions would likely be made, which should thereby be useful for predicting decision-making behavior and for supporting decision-making.

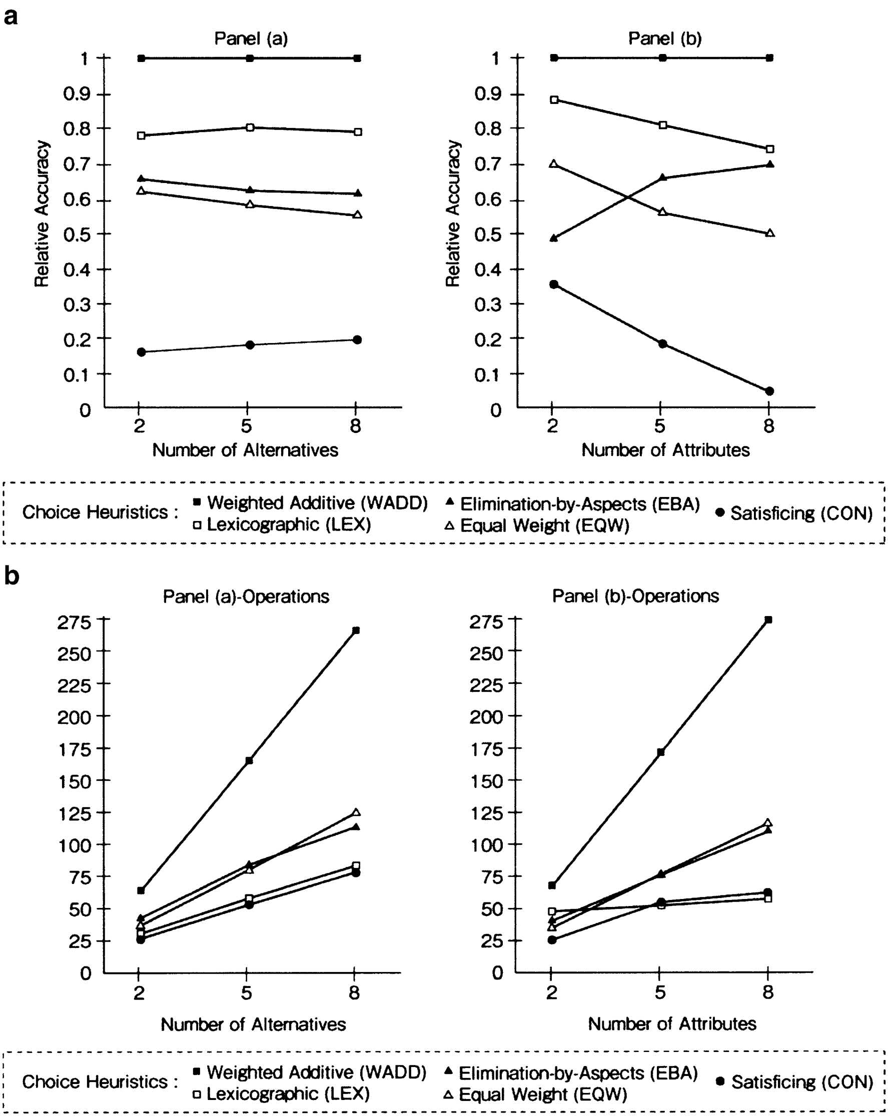

Fig. 12.4 (**a**) Effects of the number of alternatives and number of attributes on the relative accuracy of choice heuristics. *Notes*: Original reprinted by permission from Payne et al. (1990). *Source*: Payne et al. (1993). (**b**) Effects of the number of alternatives and number of attributes on the average number of operatuins of choice heuristics. *Notes*: Original reprinted by permission from Payne et al. (1990). *Source*: Payne et al. (1993)

3 Metacognitive Mechanism Model

This model, proposed by Takemura (1985, 1996b), aims at a uniform explanation of the effects of the complexity of the tasks, involvement, emotions, and other factors on the selection of decision strategies using the mechanism of metacognition. The term metacognition refers the "cognition of cognition," which, in this case, is the awareness of the state of thinking about decision-making such as "I am confused and cannot decide on one," "I am not satisfied with the decision," and "I want to make a good decision." The roles of this metacognition include monitoring of the decision-making process and allocation of processing resources to control the decision-making process. First, monitoring is the function of observing the psychological process of acknowledging a decision-making problem and implementing a selection method. The feeling of oneself being confused or the decision-making problem being difficult in the decision-making process is conceivably the result of monitoring. This monitoring is probably conducted while the decision-making problem is being cognized. This monitoring, however, is not practiced when the decision-making is typical or when the time is limited.

The metacognitive mechanism model adopts a decision strategy based on the information acquired from this monitoring, which is governed by the function of processing resource allocation. Processing resources are the resources for cognitive processing that support expectation, attention, or effort. Although selection of decision strategies is intended to allocate processing resources efficiently, completely efficient allocation of the resources is difficult to achieve when adopting a selection method. It is assumed that the process of implementing certain decision strategies by trial and error is monitored, and that the strategies are modified and adjusted until an appropriate decision strategy for efficient allocation of processing resources is identified. Figures 12.5 and 12.6 suggest that monitoring starts at the stage of mental construction of the problem at the beginning of the decision-making process, which continues through evaluation, selection of decision strategies, and implementation and even after the decision is made.

The characteristics (complexity of the task) and context (time restrictions, etc.) of decision problems and the abilities and psychological state, such as involvement, of the decision-maker are thought to be related closely to these processing resources. For example, a complex decision-making problem with large numbers of alternatives and attributes and information overload would require a large amount of processing resources for the understanding of the decision problem and other mental activities. As a consequence, the processing resources allocated at each stage of the decision-making process are conceivably reduced (see Fig. 12.5). In contrast, a high level of involvement is likely to engender cognitive elaboration, which would increase the processing resources allocated at each stage of the decision-making process (see Fig. 12.6).

Similarly, the characteristics and context of decision problems and the abilities and psychological state of the decision-maker are thought to be closely related also to monitoring. For example, the monitoring function at each stage of the

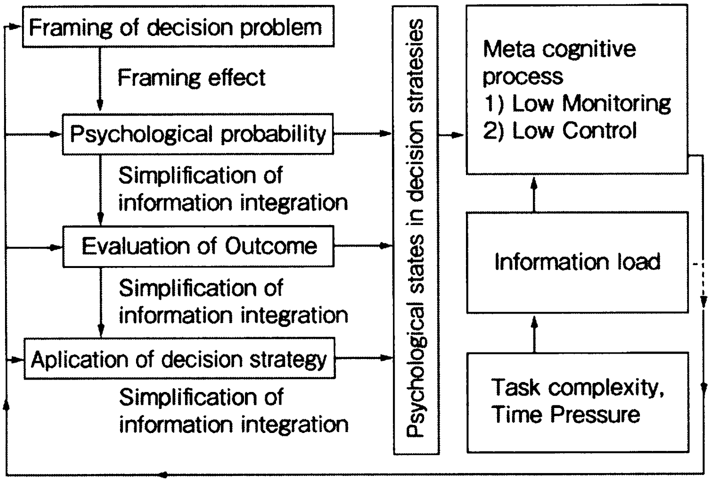

Fig. 12.5 Metacognitive model of decision making process under information overload. *Source*: Takemura (1998)

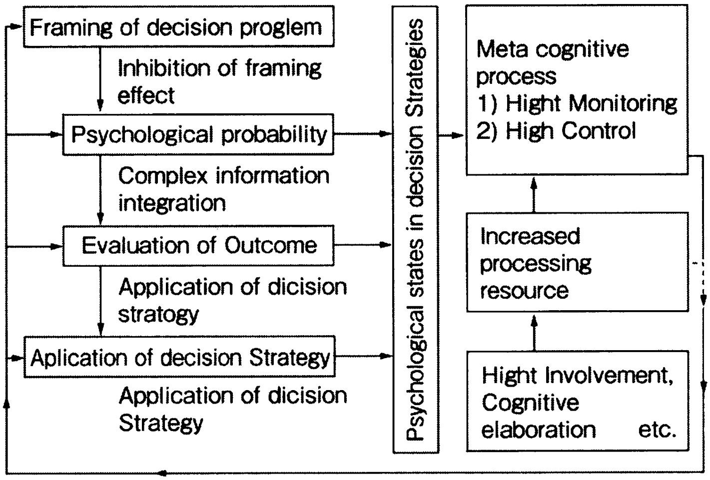

Fig. 12.6 Metacognitive model of decision making process under cognitive elaboration. *Source*: Takemura (1998)

decision-making process is likely to decline for a complex decision-making problem with large numbers of alternatives and attributes and information overload (see Fig. 12.5). Meanwhile, a high level of involvement is likely to result in cognitive elaboration, which would increase the monitoring function at each stage of the decision-making process (see Fig. 12.6).

Figure 12.5 presents the process assumed for the metacognition mechanism and decision-making in the case of a complex decision-making problem with numerous alternatives and attributes and information overload. First, when the task is complex and overloaded with information, the processing resources allocated is likely to be reduced and the monitoring function is likely to decline, which tends to cause the so-called framing effect (Tversky and Kahneman 1981) in the mental construction of the decision-making problem. The psychological probability, assessment of the result, and the form of information integration in the selection of decision strategies are expected to be simple and non-compensatory.

Figure 12.6 presents the process assumed for the metacognition mechanism and decision-making in the state resulting in cognitive elaboration. First, a high level of involvement is likely to result in cognitive elaboration, which would increase the processing resources allocated and increase the monitoring function. Consequently, the so-called framing effect (Tversky and Kahneman 1981) is controlled in the mental construction of the decision-making problem. The psychological probability, assessment, and the method of information integration in the selection of decision strategies might be compensatory, and the processing might be complex and involve repeated examination of information.

Furthermore, this metacognition mechanism model explains the problem of the effect of emotion and involvement on the decision-making process, which was presented last time (Takemura 1996b). In other words, positive emotions reduce the monitoring function of the metacognition mechanism and the processing resources to be allocated, which is, thereby, likely to result in selection of a simple decision strategy and shorter time until the decision is made. Negative emotions increase the monitoring function of the metacognition mechanism, but reduce the processing resources allocated. Therefore, a simple form of information integration would not be adopted and longer time than usual would be taken for the decision-making. Because negative emotions would increase only the monitoring function, the interaction effect because of involvement would emerge. Because positive emotions would reduce the monitoring function, the interaction effect attributable to involvement is unlikely to appear.

This metacognition mechanism model is qualitative and inappropriate for quantitative forecasting. It might be useful, however, for uniformly interpreting or explaining the effects of various factors on the selection of decision strategies.

4 Behavioral Decision Theory and Neuroeconomics

Behavioral decision theory has also been connected with the field called neuroeconomics in recent years. Neuroeconomics is the research area that is intended to integrate psychology, economics, and neuroscience. It seeks to identify appropriate models of people's selection and decision-making and explain the neuroscientific basis of decision-making phenomena using various theoretical approaches and experimental methods (Sanfey 2007a, b; Takemura et al. 2008b; Takemura 2009). The area of fusion between marketing and neuroeconomics called neuromarketing has also been developing.

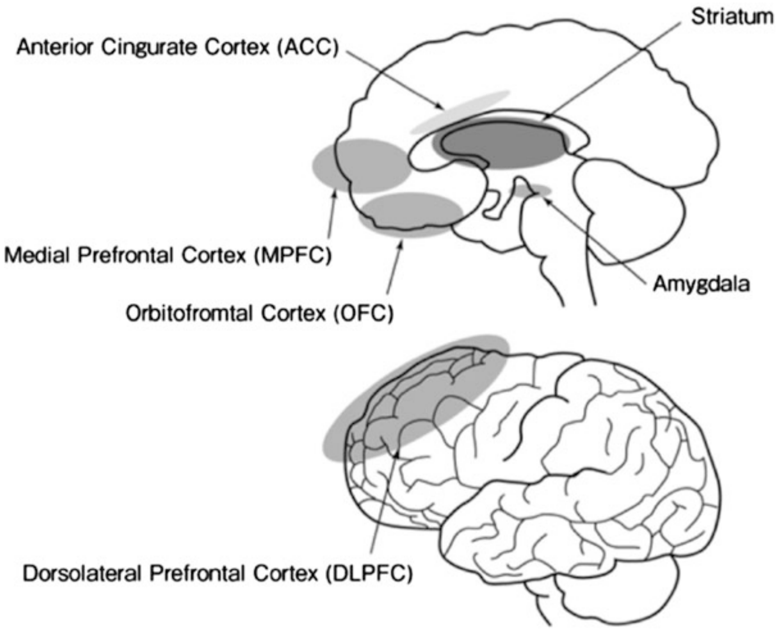

Fig. 12.7 Regions of the brain related to decision making. *Source*: Takemura et al. (2008a)

The reasons for the progress of such fields as neuroeconomics and neuromarketing are, first, that noninvasive measurement of brain activity such as functional magnetic resonance imaging (fMRI) and positron emission tomography (PET) has developed, and the system of making findings, which have been made only through the behavioral experiments done by psychologists and economists in the past, in cooperation with neuroscientists, has been established. The second reason is that, as noted earlier, the human model of a "rational economic human" assumed in economics in the past was questioned by numerous economists and psychologists. That inquiry produced behavioral decision theory and behavioral economics to describe the actual human behavior of decision-making and theorize the behavior.

The most experimentally examined theory in the studies of neuroeconomics is prospect theory. Knuston et al. (2007) performed an experiment in which an actual product was presented, and subsequently, the price was presented to the test subjects using fMRI, who were, then, to decide whether they would purchase the product. The result was that the more attractive the product, the more active the nucleus accumbens (NAcc) in the ventral striatum. When high prices were shown, the insula was active and the activity of the medial prefrontal cortex (MPFC) decreased (see Fig. 12.7). This result suggests that the brain activities in the areas of loss and gain differ and can be interpreted as being consistent with prospect theory, which uses different value functions in the areas of loss and gain.

What is important in prospect theory is that the results are assessed based on the difference from a reference point. Previous studies have revealed activity of the MPFC and NAcc when compensation is not paid against expectations rather than according to expectations (Loewenstein et al. 2008). These parts are likely to be related to the reward system, which suggests that there is the brain activity corresponding to the editing phase assumed in prospect theory such as the difference from expectations rather than the ultimate condition of a result.

Gonzalez et al. (2005) who investigated the editing phase in prospect theory, found that people's decision-making would vary depending on whether the linguistic expression of the same decision-making problem was positive or negative, and that a negative expression would tend to promote risk-taking selection. Activity of the right dorsolateral prefrontal cortex and the intraparietal sulcus was detected from fMRI when a risk-taking selection was made (Gonzalez et al. 2005).

De Martino et al. (2006) examined the decision-making that corresponded to the evaluation phase using fMRI. The risk-aversion tendency was observed when profit was to be selected, and risk-taking orientation was evident when loss was to be selected. Examination of functional brain imaging in each state of profit and loss suggested that decision-making based on emotions triggered by the amygdale was related to the reversal of preference. This study also implied that the orbitofrontal cortex and MPFC that are related to the control of decision-making

In addition, there is a research finding to suggest that a state in which the probability distribution can only be vaguely identified, rather than a situation with the risk of a known probability distribution, in decision-making has more active orbitofrontal cortex and amygdale. The ambiguity is not considered just as the expected utility theory, but rather, it is consistent with prospect theory (Loewenstein et al. 2008).

Cognitive psychological methods such as the process tracing technique might be used to learn the ambiguous decision-making process. Using neuroscientific methods is also extremely promising.

Hsu et al. (2005) compared decision-making under ambiguity and decision-making under risk and observed activation of the orbitofrontal cortex (OFC) (presumably related to the integration of emotions and cognitive input), the amygdale (which presumably reacts to emotional information), and the dorsomedial prefrontal cortex (DMPFC) (which presumably adjusts the activity of the amygdale) in decision-making under ambiguity and activation of the caudate nucleus in decision-making under risk. Hsu et al. (2005) also demonstrated that the more active the OFC, the stronger the tendency of ambiguity aversion.

Huettel et al. (2006) demonstrated, based on experiments that used secure alternatives, risky alternatives, and ambiguous alternatives that decision-making involving ambiguous alternatives had indicated activity in the posterior inferior frontal sulcus (pIFS) [included in the lateral prefrontal cortex (LPFC)], the anterior insular cortex (aINS), and the posterior parietal cortex (pPAR). Huettel et al. (2006) also estimated the parameter of ambiguity aversion for each test subject based on the results of the experiments and explained that the greater the activity of the pIFS, the stronger the tendency of ambiguity aversion.

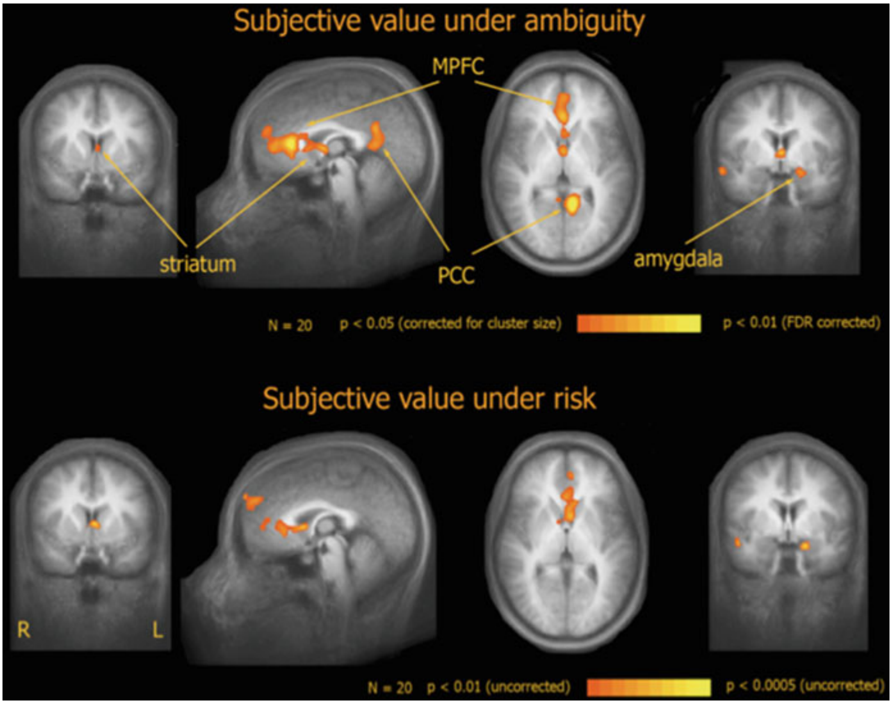

Fig. 12.8 Parts indicating subjective value and correlation associated with decision-making under ambiguity and under risk by Levy et al. *Source*: Levy et al. (2010)

Levy et al. (2010) showed through decision-making problems under ambiguity and under risk that the subjective value under ambiguity correlated with a larger number of brain parts than the subjective value under risk (see Fig. 12.8). No parts specifically related to each form of decision-making were found, however, and the striatum and MPFC were identified as the parts activated least in common.

Although not in the context of decision-making, Bach et al. (2009) performed fear conditioning with situations under risk, under ambiguity, and under ignorance as the condition stimuli and presented the posterior inferior frontal gyrus (pIFG), the right posterior parietal cortex (pPAR), and the lateral occipital cortex as the parts indicating greater activity in the situation under ambiguity in comparison to those under risk and under ignorance.

Before the studies described earlier were reported, Krain et al. (2006) conducted a neuroscientific study of decision-making using gambling tasks, in which the tasks were categorized into risky decision-making and ambiguous decision-making according to the characteristics of the tasks used; a meta-analysis was performed.

They categorized the studies with tasks that demanded selection between high-risk and low-risk alternatives as risky decision-making and those requiring selection from alternatives whose degrees of the result and probability were equivalent as ambiguous decision-making. They argued that studies using the Iowa gambling task and Cambridge risk task, which are often used in decision-making studies in conventional neuroscience, constituted studies of risky decision-making and not neuroscientific research that directly examined decision-making related to ambiguity.

They also assumed a dual-process consisting of a "hot" cognitive processes and "cool" cognitive processes as a premise of the examination, in which decision-making under risk corresponded to an emotional and intuitive "hot" process and decision-making under ambiguity corresponded to a rational and deliberate "cool" process.

On the assumption of the knowledge that the "hot" and "cool" cognitive processes corresponded to the localization of brain function between the OFC and the dorsolateral prefrontal cortex (DLPFC) and assuming it as a result of meta-analysis, decision-making under risk suggested the activity of the OFC and decision-making under ambiguity suggested the activity of DLPFC, just as they had expected.

The categorization criteria used by them, however, are not consistent with the definition of decision-making under ambiguity based on the framework of decision theory that has been discussed in this article. The tasks used in the studies categorized as ambiguous decision-making according to their criteria include one that consecutively presented screens showing a red or blue ball and asked which color had appeared more, a high-and-low game using cards from 1 to 10, one that asked whether a red or black card would appear from a stack of playing cards, rock–paper–scissors, and others. These tasks would be included in decision-making under risk in the framework discussed in this article.

For this reason, their findings might not be accepted directly. The perspective of the "hot" and "cool" cognitive processes assumed by them might provide many suggestions to studies of decision-making under uncertainty. However, it cannot be assumed to handle the difference from decision-making under risk and decision-making directly under ambiguity. In fact, those studies described earlier defined ambiguity based on strict criteria from the perspective of decision theory, which observed activity in parts that differ from those found in the meta-analysis of Krain et al. (2006). Further improvement of methods that incorporate consideration of the knowledge of decision theory is expected for the future.

As presented so far, the research of behavioral decision theory suggests that people's decision-making process is extremely situation-dependent and that various psychological processes develop. Neuroeconomic research in recent years suggests that decision-making is done through a multiplex system (Sanfey 2007a, b). Decision-making has been found to involve an automatic process that is executed half unconsciously and automatically and a controlled process consciously regulated based on thoughts. Emotional processing is considered an automatic process, whereas a cognitive process involving high-level thinking is conceived as a controlled process. The important roles played not only by a high-level cognitive process but by emotions in decision-making have already been identified in a series of studies done by the group of Damasio et al. (Bechara et al. 1994, 1996, 1999, 2000). How emotional processing responds to prospect theory has not been explained fundamentally. Therefore, it is left as a theme for future research.

5 Future of Behavioral Decision Theory

Numerous studies of behavioral decision theory in the past have proved that people's decision-making rarely follows the steps assumed by expected utility theory (Simon 1957; Abelson and Levi 1985; Gigerenzer and Selten 2001). Particularly, Simon (1957) argued that rather than the principle of maximization and optimization for selecting the best one from all available alternatives, people make decisions based on the principle of satisficing to seek an alternative that is satisfactory at a certain level because of their limited information-processing capacity. Simon pointed out that, in this sense, people only had "bounded rationality."

Simon's idea of bounded rationality was further developed into a recent study based on the paradigm of fast and frugal heuristics proposed by Gigerenzer et al. (1999) and Gigerenzer (2004). Fast and frugal heuristics (cognitive rules of thumb) is a method of making adaptive decisions in the actual environment based on the minimum time, knowledge, and calculation required. Gigerenzer and Goldstein (1996), for instance, exemplified the use of computer simulation: when determining which of two cities had a larger population, recognition heuristics that would make the decision based only on the familiarity with the cities would still be relatively appropriate. This finding suggests that relatively appropriate decisions can be made even based on name recognition, by which a purchase is made because the brand is known. They have demonstrated in various cases that even fast and frugal heuristics would allow relatively appropriate decision-making. Studies based on this paradigm include the explanations of Hirota et al. (2002) and Nakamura (2004).

They proposed the idea of a priority heuristic as a fast and frugal heuristic and demonstrated that various decision-making phenomena could be explained solely by the assumption that most decisions were made simply based on one reason (Brandstätter et al. 2006). They have questioned the basic assumption of decision theory used in expected utility theory and prospect theory that people make decisions by integrating the utility and value of results and probability. Such a point is shared in common with the contingent focus model proposed by Fujii and Takemura, which assumes a simple and unitary process of decision-making through the mechanism of attention, which does not separate the probability weighted function and value function in prospect theory. Their alternative models suggest a paradigm of experimental research that differs from the conventional expected utility theory and prospect theory. Future behavioral decision theory must be developed in concert with neuroeconomic research in the paradigm based on new models other than prospect theory.

Whereas the paradigm of fast and frugal heuristics assumes that people's decision-making goes through serial information processing, a new trend in decision-making research is based on connectionist models that assume parallel information processing of people's decision-making. A connectionist model is an approach to understanding human cognitive mechanism using a network of simple processing units that considers brain neurons, which are used with almost identical meaning as in the parallel distributed processing model and neural network model (Tsuzuki and Asakawa 2003). Decision-making research through this approach ranges widely in its variety (Mori et al. 2001; Tsuzuki and Asakawa 2003; Busemeyer and Johnson 2004); major studies include decision field theory proposed by Busemeyer and Townsend (1993). Decision field theory is an attempt to explain diverse phenomena associated with decision-making process using such tools as mathematical models and computer simulation, for which a new revision has been proposed (Roe et al. 2001; Busemeyer and Johnson 2004).

Although research on the cognitive process of decision-making has thus been elaborated, research on the social psychological process of decision-making has also advanced in recent years. Schwartz (2004) and Schwartz et al. (2002) socio-psychologically extended Simon's theory and identified the relations with a sense of happiness and clinical adaptation through investigation. Schwartz et al. (2002) developed a psychological scale using a questionnaire titled "Regret and Maximization Scale." They designated a person who used a conjunctive decision strategy based on the satisficing criteria of Simon (1957) a "satisficer" and designated a person who collected as much information related to the alternatives as possible to select the best alternative as a "maximizer" and studied the psychological tendencies of the two. The results revealed positive correlations between the maximizer's tendencies and depression, perfectionism, and the level of regret and negative correlations with a sense of happiness, optimism, satisfaction with living, and self-esteem. According to Schwartz (2004), the "maximizer" regrets the alternatives that have not been selected. Therefore, such a person is less satisfied with the decision than the "satisficer" is. Such a tendency was also identified in the study done by Hisatomi et al. (2005) examining Japanese people throughout the Japanese islands from the north to south. The research results obtained by Schwartz suggest that a rational economic actor who makes optimal decisions through consideration of as many alternatives as possible is not clinically suitable in modern society. The suggestions that can be inferred from such a finding are extremely interesting.

Finally, approaches that might be beneficial in the future include, although they are not new, the research paradigm of the selection behavior of animals that might be integrated with modern behavioral decision theory (Hernstein 1961; Mazur 1998; Sakagami 1994, 1997) because, if animals' selection behavior and human decision-making are uniformly explained, behavioral decision-making research could bring about great development.

In the studies of selection behavior, the matching law—which states that the rate of selection response and the number of reinforcers obtained from the responses correspond mutually—is said to hold true generally, and many studies have been done (Hernstein 1961; Mazur 1998; Sakagami 1994, 1997). The generalized matching law developed by expanding the original matching law has also been proposed (Baum 1974, 1979).

Both the matching law and generalized matching law allow the prediction of selection phenomena that differ from the forecasts based on expected utility theory.

Some theoretical studies (Rachlin et al. 1986; Sakagami 1994) claim that prospect theory (Kahneman and Tversky 1979) can be deduced based on this generalized matching law.

Although many arguments have been made of the reasons why the matching law and generalized matching law can be established, no adequate theoretical conclusion has been reached (Mazur 1998; Sakagami 1997). As a theory to explain the matching law, the theory of melioration (Hernstein and Vaughan 1980; Vaughan 1981) has been proposed. The theory holds that animals make adjustments successively to increase either or both of time and effort for better alternatives and make selections to equalize the strengthening efficiency gained among alternatives. Some theoretical studies (Fujii and Takemura 2001, 2002; Takemura and Fujii 2004) state that the generalized matching law can be derived from the psychophysical function expressed with the logarithmic function of Fechner (1860) and the random utility theory of Thurstone (1927). Such a research paradigm of selection behavior is expected to engender more interesting findings when combined with experiments incorporating neuroscientific information such as cognitive psychological experiments on human decision-making and neuroeconomics or when examined in terms of relevance with different behavioral decision theories.

Behavioral decision theory that has thus far been described briefly might be combined more with fields such as neuroeconomics, neuromarketing, behavioral economics, and experimental psychology in the future. The research objective of behavioral decision theory to explain people's decision-making process is likely to bring about interactions with various research fields and affect not only descriptive research but normative and prescriptive studies.

6 Future Philosophical Issues Related to Behavioral Decision Theory

The B.C. philosopher Aristotle (trans. 1971) developed his ethics from the perspective of what kinds of decision-making would be appropriate. His book, the Nicomachean Ethics, begins with this line: "Every skill and every inquiry, and similarly every action and rational choice, is thought to aim at some good (Agathon); and so the good has been aptly described as that at which everything aims." The Nicomachean Ethics, as the name implies, is a book on ethics. Aristotle purportedly wrote this book in support of good decision-making.

The viewpoint of Aristotle is necessary also in modern decision theory. Contemplating what constitutes good decision-making belongs to the field of normative decision theory in decision-making theory. In the past, good decision-making has often been considered only in view of rationality. Taking norms into consideration, however, "what is good" must be defined. Such an issue has been contemplated in various ways by ethicists in the past, which must be considered also in decision theory. Along with this, behavioral decision theory is useful to examine whether people's decision-making deviates from this "good."

"Good decision-making" must be observed from a pluralistic perspective. First, decision-making should be conducted based on facts and should take rational and convincing procedures, which is also suggested by the conventional normative decision theory. It should be provided with consistency and completeness if possible. Secondly, it is important that decision-making brings happiness to people. Even if it satisfies procedural rationality, decision-making without achieving people's happiness could not be good decision-making. Achievement of happiness is unquestionably an important constituent of good decision-making. Thirdly, moral correctness is required of decision-making. Even if decision-making satisfies procedural rationality and helps individuals achieve happiness, morally wrong decisions could not be good ones. For example, decision-making that serves the happiness of the decision-makers only while hindering the wellbeing of others could not be good. Fourthly, beauty and virtue are important for good decision-making.

Even if decisions which are made satisfy the procedural rationality, bring happiness to individuals, and are morally correct, actions and decisions that are not regarded as beautiful could be regarded as only slightly as good decision-making. Take the case of splitting a bill with friends at a restaurant as an example. In view of moral fairness, happiness of individuals, and rationality, splitting the bill to the last cent and settling the payment at the restaurant would not pose any problem. Many people, however, might feel that splitting to the last cent would not be "beautiful." Good decision-making should include the element of beauty. Beauty might also be expressed with integrity, bravery, dignity, and other qualities. The final dimension of beauty should take an important position in a decision-making study. In the study of behavioral decision-making, the issue to be examined is what kinds of decision-making are regarded as beautiful by people.

In decision-making, an alternative that is dominant in all these dimensions would be unquestionably desirable, which, however, is a highly unlikely situation in reality. In decision-making in reality, there might be no dominant alternative and the composition of the alternatives does not even satisfy the comparability in many cases. Under such circumstances, people are forced to take a multidimensional approach to problem solving while simultaneously considering several aspects. Investigating how people make decisions with such pluralistic value will be a highly important research theme also for behavioral decision theory. Furthermore, the knowledge of behavioral decision theory will have an extremely important effect on normative decision theory and prescriptive decision theory.

References

Abelson, R. P., & Levi, A. (1985). Decision making and decision theory. In G. Lindzey & E. Aronson (Eds.), *The handbook of social psychology* (3rd ed., Vol. 1, pp. 231–309). New York: Random House.

Adelbratt, T., & Montgomery, H. (1980). Attractiveness of decision rules. *Acta Psychologica, 45*, 177–185.

Aristotle (1971). *Nikomakosu Rinrigaku* [Trans. by S. Takada (Ed.), *Nichomachean ethics*]. Tokyo: Iwanami Shoten.

Bach, D. R., Seymour, B., & Dolan, R. J. (2009). Neural activity associated with the passive prediction of ambiguity and risk for aversive events. *Journal of Neuroscience, 29*(6), 1648–1656.

Baum, W. M. (1974). On two types of deviation from the matching law: Bias and undermatching. *Journal of Experimental Analysis of Behavior, 22*, 231–242.

Baum, W. M. (1979). Matching, undermatching, and overmatching in studies of choice. *Journal of experimental Analysis of Behavior, 32*, 269–281.

Beach, L. R., & Mitchell, T. R. (1978). A contingency model for the selection of decision strategies. *Academy of Management Review, 3*, 439–449.

Bechara, A., Damasio, A. R., Damasio, H., & Anderson, S. W. (1994). Insensitivity to future consequences following damage to human prefrontal cortex. *Cognition, 50*, 7–15.

Bechara, A., Damasio, H., Damasio, A. R., & Lee, G. P. (1999). Different contributions of the human amygdala and ventromedial prefrontal cortex to decision-making. *Journal of Neuroscience, 19*, 5473–5481.

Bechara, A., Tranel, D., & Damasio, H. (2000). Characterization of the decision-making deficit of patients with ventromedial prefrontal cortex lesions. *Brain, 123*, 2189–2202.

Bechara, A., Tranel, D., Damasio, H., & Damasio, A. R. (1996). Failure to respond autonomically to anticipated future outcomes following damage to prefrontal cortex. *Cerebral Cortex, 6*, 215–225.

Brandstätter, E., Giggerenzer, G., & Hertwig, R. (2006). The priority heuristic: Making choices without tradeoffs. *Psychological Review, 113*, 409–432.

Busemeyer, J. R., & Johnson, J. G. (2004). Computational models of decision making. In D. J. Koehler & N. Harvey (Eds.), *Blackwell handbook of judgment and decision* (pp. 133–154). Malden: Blackwell.

Busemeyer, J. R., & Townsend, J. T. (1993). Decision field theory: A dynamic cognition approach to decision making. *Psychological Review, 100*, 432–459.

De Martino, B., Kumaran, D., Seymour, B., & Dolan, R. J. (2006). Frames, biases, and rational decision-making in the human brain. *Science, 313*, 684–687.

Fechner, G. T. (1860). *Elemente der psychophysik*. Leipzig: Breitkopf und Härtel (Reprinted, Bristol: Thoemmes Press, 1999).

Fujii, S., & Takemura, K. (2001). Fuehina no hosoku to randamu koyo riron ni motozuku ippan taio hosoku no rironteki setsumei [Theoretical explanation of generalized matching law based on fechner law and random utility theory]. *Paper presented at the 65th Conference of the Japanese Psychological Association*, 580.

Fujii, S., & Takemura, K. (2002). Fechner law and random utility hypothesis leads the matching law. *Paper presented at Society for Judgment and Decision Making*. Kansas City, MO, USA.

Gigerenzer, G. (2004). Fast and frugal heuristics: The tools of bounded rationality. In D. J. Koehler & N. Harvey (Eds.), *Blackwell handbook of judgment and decision making* (pp. 62–88). Malden: Blackwell.

Gigerenzer, G., & Goldstein, D. (1996). Reasoning the fast and frugal way: Models of bounded rationality. *Psychological Review, 103*, 650–669.

Gigerenzer, G., & Selten, R. (Eds.). (2001). *Bounded rationality: The adaptive toolbox*. Cambridge: MIT.

Gigerenzer, G., Todd, P. M., & The ABC Research Group (Eds.). (1999). *Simple heuristics that make us smart*. New York: Oxford University Press.

Gonzalez, C., Dana, J., Koshino, H., & Just, M. (2005). The framing effect and risky decisions: Examining cognitive functions with fMRI. *Journal of Economic Psychology, 26*, 1–20.

Herrnstein, R. J. (1961). Relative and absolute strength of response as a function of frequency of reinforcement. *Journal of Experimental Analysis of Behavior, 4*, 267–272.

Hernstein, R. J., & Vaughan, W., Jr. (1980). Melioration and behavioral allocation. In J. E. R. Staddon (Ed.), *Limits to action: The allocation of individual behavior* (pp. 143–176). New York: Academic.

Hirota, S., Masuda, S., & Sakagami, T. (2002). *Shinrigaku ga egaku risuku no sekai: Kodoteki ishikettei nyumon [The world of risk drawn by psychology: An introduction to behavioral decision-making]*. Tokyo: Keio University Press.

Hisatomi, T., Isobe, A., Oniwa, T., Matsui, Y., Ui, M., Takahashi, N., & Takemura, K. (2005). Anshin to fuan no shakai shinri (IV): Ishikettei sutairu to shinrai tono kanrensei [Social psychology of security and insecurity (IV): Relevance between decision-making style and trust]. *Paper presented at the 46th Conference of the Japanese Society of Social Psychology* (pp. 230–231).

Hsu, M., Bhatt, M., Adolphs, R., Tranel, D., & Camerer, C. F. (2005). Neural systems responding to degrees of uncertainty in human decision-making. *Science, 310*, 1680–1683.

Huettel, S. A., Stowe, C. J., Gordon, E. M., Warner, B. T., & Platt, M. L. (2006). Neural signatures of economic preferences for risk and ambiguity. *Neuron, 49*, 765–775.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47*, 263–292.

Knuston, B., Rick, S., Wimmer, G., Prelec, D., & Loewenstein, G. (2007). Neural predictors of purchases. *Neuron, 53*, 147–156.

Kobashi, Y. (1988). *Ninchi kagaku sensho 18 kettei wo shien suru [Selected works in cognitive science 18: Support decisions]*. Tokyo: University of Tokyo Press.

Krain, A. L., Wilson, A. M., Arbuckle, R., Castellanos, F. X., & Milham, M. P. (2006). Distinct neural mechanisms of risk and ambiguity: A meta-analysis of decision-making. *Neuro Image, 32*, 477–484.

Levy, I., Snell, J., Nelson, A. J., Rustichini, A., & Glimcher, P. W. (2010). Neural representation of subjective value under risk and ambiguity. *Journal of Neurophysiology, 103*, 1036–1047.

Loewenstein, G., Rick, S., & Cohen, J. (2008). Neuroeconomics. *Annual Review of Psychology, 59*, 647–672.

Mazur, J. E. (1998). *Learning and behavior* (4th ed.). Englewood Cliffs: Prentice Hall [Translated into Japanese by Iso, H., Sakagami, T., & Kawai, N. (1999). *Meiza No Gakushu To Kodo* Japanese Second ed. Hiheisha].

Montgomery, H. (1983). Decision rules and the search for a dominance structure: Towards a process model of decision making. In P. C. Humphreys, O. Svenson, & A. Vari (Eds.), *Analyzing and aiding decision processes* (pp. 343–369). Amsterdam: North-Holland.

Montgomery, H. (1993). The search for a dominance structure in decision making: Examining the evidence. In G. A. Klein, J. Orasanu, R. Calderwood, & C. E. Zsambok (Eds.), *Decision making in action: Models and methods* (pp. 182–187). Norwood: Ablex.

Mori, K., Tsuzuki, T., & Kusumi, T. (2001). *Konekushonisuto moderu to shinrigaku: Nou no shimyureshon ni yoru kokoro no rikai [Connectionist model and psychology: Understanding mind through brain simulation]*. Kyoto: Kitaohji Shobo.

Nakamura, K. (2004). Kosoku kenyaku hyurisutikku no shinrigakuteki datosei wo meguru jisshoteki, gainenteki giron no doko [Trends in empirical and conceptual argument on the psychological propriety of fast and frugal heuristics]. *Japanese Psychological Review, 47*, 453–477.

Payne, J. W., & Bettman, J. R. (2004). Walking with the scarecrow: The information-processing approach to decision research. In D. J. Koehler & N. Harvey (Eds.), *Blackwell handbook of judgment and decision* (pp. 110–132). Malden: Blackwell.

Payne, J. W., Bettman, J. R., & Johnson, E. J. (1993). *The adaptive decision maker*. Cambridge: Cambridge University Press.

Payne, J. W., Johnson, E. J., Bettman, J. R., & Coupey, E. (1990). Understanding contingent choice: A computer simulation approach. *IEEE Transactions on Systems, Man, and Cybernetics, 20*, 303.

Rachlin, H., Logue, A. W., Gibbon, J., & Frankel, M. (1986). Cognition and behavior in studies of choice. *Psychological Review, 93*, 33–45.

Roe, R., Busemeyer, J. R., & Townsend, J. T. (2001). Multi-alternative decision field theory: A dynamic connectionist model of decision-making. *Psychological Review, 108*, 370–392.

Sakagami, T. (1994). Fukakujitsusei wo meguru dobutsu kodo kenkyu [Study of animal behavior concerning uncertainty]. *Japanese Psychological Review, 37*, 294–323.

Sakagami, T. (1997). Kodo keizaigaku to sentaku riron [Behavioral economics and selection theory]. *Japanese Journal of Behavior Analysis, 11*, 88–108.

Sanfey, A. G. (2007a). Decision neuroscience: New directions in studies of judgment and decision making. *Current Directions in Psychological Science, 16*, 151–155.

Sanfey, A. G. (2007b). Social decision making: Insights from game theory and neuroscience. *Science, 318*, 598–602.

Schwartz, B. (2004). *The paradox of choice: Why more is less*. New York: HarperCollins [Translated into Japanese by Mizuho, N. (2004). *Naze Erabu Tabi Ni Kokai Surunoka: 'Sentaku No Jiyu' No Otoshiana*. Tokyo: Random House Kodansha].

Schwartz, B., Ward, A., Monterosso, J., Lyubomirsky, S., White, K., & Lehman, D. R. (2002). Maximizing versus satisficing: Happiness is a matter of choice. *Journal of Personality and Social Psychology, 83*, 1178–1197.

Simon, H. A. (1957). *Administrative behavior: A study of decision making process in administrative organization*. New York: McMillan.

Takemura, K. (1985). Ishikettei sutorateji jikko ni okeru meta ninchi katei moderu [Metacognition process model in the implementation of decision-making strategy]. *Doshisha Psychological Review, 32*, 16–22.

Takemura, K. (1996a). Ishikettei to sono shien [Decision-making and its support]. In S. Ichikawa (Ed.), *Ninchi shinrigaku 4kan shiko [Cognition psychology, vol. 4: Thoughts* (pp. 81–105). Tokyo: University of Tokyo Press.

Takemura, K. (1996b). *Ishikettei no shinri: Sono katei no tankyu [Psychology of decision-making: Inquiry into its process]*. Tokyo: Fukumura Shuppan.

Takemura, K. (1998). *Ishi kettei no shinri [Psychology of decision-making]*. Tokyo: Fukumura Shuppan.

Takemura, K. (2009). Ishikettei to shinkei keizaigaku [Decision-making and neuroeconomics]. *Japanese Journal of Clinical Psychiatry, 38*, 35–42.

Takemura, K. & Fujii, S. (2004). Interpretation of matching law: From the assumptions of Luce choice axiom, Fechner law, and random utility maximization. *Paper presented at the 35th European Mathematical Psychology Group Meeting, Ghent Belgium.*

Takemura, K., Ideno, T., Ohkubo, S., & Matsui, H. (2008a). Shinkei keizai gaku to zentoyo [Neuroeconomics and frontal lobe]. *Bunshi seishin igaku [Japanese Journal of Molecular Psychiatry], 8*(38), 122.

Takemura, K., et al. (2008b). Shinkei keizaigaku to zentoyo [Neuroeconomics and the frontal lobe]. *Bunshi Seishin Igaku, 8*, 119–124.

Thurstone, L. (1927). A law of comparative judgment. *Psychological Review, 34*, 273–286.

Tsuzuki, T., & Asakawa, S. (2003). Ninchi kagaku to konekushonisuto moderu [Cognitive science and the connectionist model]. *Cognitive Studies, 10*, 2–8.

Tversky, A., & Kahneman, D. (1981). The framing of decisions and the psychology of choice. *Science, 211*, 453–458.

Vaughan, W., Jr. (1981). Melioration, matching, and maximization. *Journal of Experimental Analysis of Behavior, 36*, 141–149.

Part VII
Behavioral Decision Theory and Good Decision Making

Chapter 13
Behavioral Decision Theory and Good Decision Making

The final chapter of this book presents a critical examination of the psychological models of multi-attribute decision-making, findings obtained from them, and rational decision-making and considers what constitutes a "good decision." First, a basic framework for ordinal utility theory based on Takemura (2011a, b) is presented as normative analysis and is examined in view of rationality. By subsequently defining the version of ordinal utility theory expanded to multi-attribute decision-making, we will re-interpret the rationality of multi-attribute decision-making based on Arrow's general possibility theorem. Re-interpretation of the general possibility theorem of Arrow (1951) suggests that the rational multi-attribute decision-making defined here could not be performed with the exception of one-dimensional decision-making based only on specific attributes. We descriptively analyze people's multi-attribute decision-making to demonstrate, based on the psychological model of decision-making, the tendency of people to use one-dimensional decision-making to solve issues of multi-attribute decision-making. Finally, prescriptive examinations of multi-attribute decision-making are performed to support the argument that decision making from a pluralistic perspective results in a "good decision" even though one-dimensional decision-making should be avoided and even though rationality in the above sense might not be satisfied, particularly in important decision-making.

1 Multi-Attribute Decision Making and Best Decision

1.1 Best Decision

We make various decisions in our daily life. We make decisions on every occasion: from casual ones such as what to eat for lunch to more serious decisions such as an individual's future course and government policy. Decision-making, therefore, becomes a rather important concept when considering practical activities and making choices in life.

K. Takemura, *Behavioral Decision Theory: Psychological and Mathematical Descriptions of Human Choice Behavior*, DOI 10.1007/978-4-431-54580-4_13,

As described in Chap. 12, Aristotle (trans. 1971) in the Nicomachean Ethics purportedly seeks the highest good (*agathon*) in people's act of selecting. This can be made more comprehensible by asking and answering the following questions: Why do people wish to go to a good college? Because they are more likely to get a good job. Why do they wish to get a good job? Because they wish to live a good life. Why do they wish to live a good life? Because they seek something good. This good thing might be the highest good. One might consider another example. We assume that of a person who is going to purchase a car. He has decided to buy a car with a brand name X sold by Company A. Why does he purchase the X-branded car from Company A? First, he considered the high durability and resale value of cars from Company A. Cars of the X brand are fuel-efficient and are presumably environmentally friendly for being hybrid cars. He also liked the design. The price is lower than that of similar cars from other automakers. He was also given a large discount. He also noted that the salesperson treated him well. These factors suggest that people make decisions for various and complex reasons. We can think about why the high fuel economy and likable design are positive factors and what implications that environmental friendliness might have. Good gas mileage and design are appreciated possibly for the satisfaction added to the life of the car buyer as a consumer.

A better environment meanwhile might be associated with welfare of both the car buyer and other people. If we seek more factors that increase the value, we might arrive at the highest good. Examining optimal decision-making reveals that rationality is necessary in the course of making the decision. A decision for which the purpose and method are contradictory is somehow not right. When we intend to make a good decision, it appears that we often assume that we will select the optimal alternative, i.e., the best decision-making. This is what is called the "best decision-making" in the world of business.

1.2 Multi-Attribute Decision-Making

Decision-making in many cases has a multi-attribute structure. Consider the example of multi-attribute decision-making in the selection of a personal computer presented in Fig. 13.1 (Okubo and Takemura 2011). In the purchase decision making to select a PC from Brands A, B, and C, we assume that the consumer makes the decision by comparing two brands to select one with more preferable attributes than the other and that the consumer purchases the one that remains in the end. Such a means of making a decision is often witnessed in the technique called monitoring information acquisition, which we use to analyze the process of obtaining information, and a verbal protocol for making verbal reports. It might seem that the option that will ultimately prevail would be the same irrespective of the order of selection. If we follow this procedure and compare Brands A and B first, Brand A prevails over Brand B in the price and performance whereas Brand C outscores Brand A in the performance and design in the comparison between Brands A and C. If the comparison begins with Brands B and C, however, Brand B would remain after the final

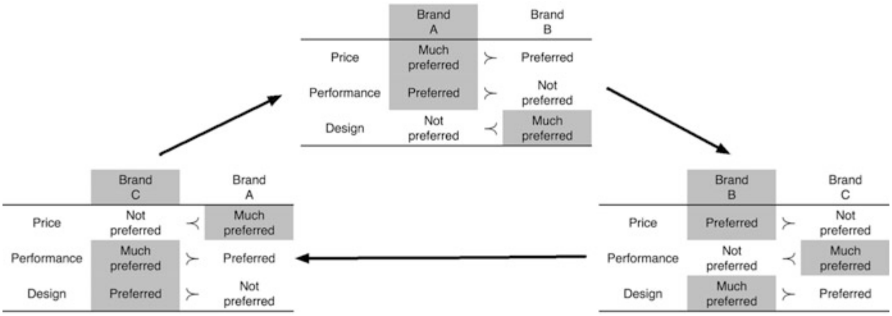

Fig. 13.1 Example of cyclic preference relation that does not satisfy transitivity. *Source*: Okubo and Takemura (2011)

stage. Despite the preference for Brand A to Brand B and Brand B to Brand C, therefore, the transitivity by which Brand A is preferred to Brand C is not satisfied. Instead, the result is the reverse relation in which Brand C is selected over Brand A (see Fig. 13.1).

1.3 Difficulty of Multi-Attribute Decision-Making and Its Psychological Cause

Under such circumstances, information related to the order in which the brand options are focused suggests the final result of the selection. Although Fig. 13.1 exemplifies only simulated conditions, this implies that the result of selection could not be predicted without knowing the order of processing information in the decision making process even when it is based on the same decision-making standard. Our past research findings related to decision-making process indicate that people's decision-making process, in fact, relies on the path. Such reliance on the circumstances or paths increases along with a larger number of options, and evidently, also with increased emotion or excitement.

Such a phenomenon hints that people's actual preference relations do not satisfy the properties of weak orders (properties that satisfy both transitivity and connectedness) assumed in the expected utility theory and multi-attribute utility theory described later. This inference further suggests that people are incapable of making the best decision and of maximizing the utility.

Studies of good decision-making belong to the field called "normative decision theory." Conventional normative decision theory often assesses "goodness" in view of the rationality of form. This chapter, too, first examines decision-making from a pluralistic perspective based on the formal rationality in this sense. Normally, theories to consider decision-making from a pluralistic perspective include the system of the so-called multi-attribute utility theory. This theoretical system concerns the method of deriving the eventual decision by trading off the value in multiple dimensions. Although such a concept is extremely important, this book first develops an argument from a perspective that is slightly different from the ordinary multi-attribute utility theory. In other words, rather than considering the tradeoffs of various values as the starting point of the argument, we first perform a normative analysis from the perspective that no values can be compared. Although this assumption might seem somewhat unnatural, it has certain significance as the starting point of the argument, considering the difficulty of trading off values and determining values of higher importance in the actual decision-making. For instance, the question of whether respect for a human life or achievement of justice is more important cannot be answered easily. Trading off one for the other would be considerably difficult because some decision of an organization is necessary even though some value tradeoffs would ultimately be necessary.

Although acknowledging the importance of the tradeoff issue, this chapter first develops the argument without assuming the possibility of tradeoffs from the beginning. This is similar to the idea of not assuming the comparison of utility among individuals as the starting point of an argument. There is also a method of normative analysis that uses game theory by setting up multiple subjects to examine the interaction among the subjects of decision-making. This chapter, however, does not perform the analysis from this type of perspective. Although the perspective of game theory allows interesting studies as to whether honest expression of preference is rational and whether individual rationality engenders collective rationality, this chapter uses an extremely simple view that decision-making of both individuals and a group seemingly involves general intentions. Examination based on such assumption is likely to facilitate a general understanding of multi-attribute decision-making despite the probable limitation posed by the simplification of the issue.

This chapter exemplifies the perspective of form that, when multi-attribute decision-making is viewed from the perspective described above, it satisfies such rationality standards as transitivity and connectivity and conditions considered appropriate in multi-attribute decision-making contradict. This can be derived by application of and re-interpreting the mathematical structure of the general possibility theorem of group decision-making presented by Arrow (1951) to the multi-attribute decision-making defined above. Applying Arrow's general possibility theorem on this assumption results in the finding that rational decision-making is possible only when it is based on one-dimensional standards, which suggests that rational decisions generally cannot be made if the pluralistic values cannot be ranked, which means that making the best decision would also be meaningless. Although the actual decision-making involves the issue of such unfeasibility of decision-making, it appears that we do not normally face the unfeasibility in the psychological aspect.

The question of whether people psychologically make their problems one-dimensional to avoid the confrontation with the unfeasibility of decision-making will be examined based on the theory of "mental ruler" proposed by the author. According to this theory, people reduce the difficulty of decision-making by one-dimensionally viewing a multidimensional problem. Making an issue one-dimensional reduces psychological burden in some respects and encourages more efficient decision-making, which, however, involves the risk of ignoring the attributes of important decision-making. This chapter claims that, from prescriptive view, one-dimensional criteria of decision-making should be avoided. If the result of decision-making is particularly important, decision based on one value only might be risky. To make a good decision, it is important to make the decision comprehensively by particularly addressing the plurality of values. It should be pointed out that pluralistic value must be considered based on a prescriptive view and originally a normative view, and various values must be combined or traded off even though such practice is extremely difficult. This discussion is intended to reveal the conditions that are likely to require judgment based a one-dimensional "mental ruler." The argument will add at the end how to make a decision for which such one-dimensional judgment is not necessary.

2 Weak Order in Multi-Attribute Decision-Making and Additive Conjoint Structure

2.1 Making the Best Decision with a Single Attribute

Before we consider multi-attribute decision-making, we will review single-attribute decision-making. As discussed in Chap. 2, the theorem for a weak order of preference indicates the conditions for the best decision-making and conditions for utility maximization.

2.1.1 Theorem for a Weak Order on a Finite Set (Krantz et al. 1971)

If the preference structure of a finite nonempty set, $\langle A, \succsim \rangle$, is a weak order, then there exists a real-valued function u on A ($u : A \rightarrow Re$) such that,

$$\forall x, y \in A, x \succsim y \Leftrightarrow u(x) \geq u(y).$$

In other words, this theorem means that if the preference made is a weak order, it can be expressed with a function that takes real numbers that maintain the preference relation. Therefore, this indicates that the preference relation of a qualitative weak order can be examined by quantifying it using ordinal utility.

2.1.2 Uniqueness Theorem for a Weak Order on a Finite Set (Krantz et al. 1971)

If the preference structure $\langle A, \succsim \rangle$ of a finite nonempty set A is a weak order and only in such a case, $\langle A, \succsim \rangle$ is expressed by $\langle \mathrm{Re}, \geq \rangle$ through the real-valued function $u : A \rightarrow Re$ indicated in the above theorem, and the structure $\langle \langle A, \succsim \rangle, \langle \mathrm{Re}, \geq \rangle u \rangle$ becomes an ordinal scale. Although this theorem assumes a finite set, we know that it also applies to countably and uncountably infinite sets (Krantz et al. 1971). In this sense, if the preference satisfies connectivity and transitivity, preference or equality of an arbitrary pair of alternatives can be expressed, then at least one best option can be selected, and this preference is equivalent to utility maximization. Although the best selection can be made if the preference is acyclic even if it is not a weak order, being a weak order guarantees the maximization of utility. This is true not only in the case of a single attribute, but in the preference relation of alternatives in multi-attribute decision-making. This can be described as an additive conjoint system of multi-attribute decision-making.

2.2 *Multi-Attribute Decision-Making and Additive Conjoint Structure*

The types of analysis that assumes a preference weak order include conjoint analysis, which assumes the additivity of the operation to totalize the utility values of attributes. Conjoint analysis is often used particularly for understanding preference for marketing purposes. In new product development, for example, it is used to find out which attribute value of an existing product should be changed to produce a new product that can attract consumers the most. It is also used to calculate the market share of the new product through simulation. Currently, conjoint analysis is used most frequently in marketing. It can also be applied to studies of preference judgment such as a survey of high school students in their selection of a college to attend. Not only for preferences, it is also used for studies of risk assessment by civil engineering experts, and its potential applicability to other fields is markedly high.

As indicated in the pioneering study of Luce and Tukey (1964), conjoint analysis is an analytical technique originally developed in the field of mathematical psychology to compose an additive real-valued function (additive utility function) that is equivalent to the interval scale from preference data at the level of ordinal scale (or more precisely, a scale that satisfies weak ordering). We know that the preference relation must satisfy a group of axioms to compose such an additive real-valued function.

Initial utility estimation based on conjoint analysis in many cases was affected by studies in view of such axioms, which assumed an ordinal scale for the respondents' preference judgment and used monotonic transformation methods such as monotone analysis of variance (MONANOVA) for the estimation [e.g., (Shepard et al. 1972)]. In recent years, however, conjoint analysis using ordinary least squares based on dummy variables has been used more frequently (Louviere 1988; Cattin and Wittink 1989). While conjoint analysis using ordinary least squares requires, to be exact, that the preference judgment is at or above the interval scale level, simulation results of studies have shown that the result will be similar to MONANOVA when monotonic transformation is performed on the assumption of an ordinal scale (Carmone et al. 1978).

The following briefly describes the conjoint analysis that uses least squares contained in ordinary statistical packages. Conjoint analysis uses the following linear model to express the response (assessment result) r_i of a particular respondent to the subject of assessment.

$$r_i = \sum_{j=1}^{q} u_j\left(k_{ji}\right)$$

Where $u_j(k_{ji})$ is the utility (partial utility) of element (attribute) j of the assessment subject i at the level k_{ji} (simplified as u_{jk} in the following).

The method of calculation for partial utility function u_j in the estimation of assessment result r_i varies depending on whether the case is (1) discrete elements for which a relation such as a linear equation and quadratic expression cannot always be assumed between each level of the elements, (2) linear elements for which a linear relation can be assumed between each level, or (3) quadratic function factors [either with an ideal point (ideal factor) or anti-ideal point (anti-ideal factor)] for which a quadratic function relation can be assumed between each level. For linear elements, for instance, the estimated assessment value changes with the linear function of level values, and for quadratic function factors, the estimated assessment value changes with the quadratic function of level values. We will estimate u_{jk} by specifying these functions.

When obtaining the actual data, we will present all profiles of assessment subjects to the respondents and collect the assessment scores and ranking data. However, an increased number of attributes considered and attribute levels will make it more difficult for the respondents to make assessments through ranking and other means. We will need various measures to reduce their burden. The number of profiles presented to the respondents is often reduced using an orthogonal design.

The decision maker must have the axiomatic property of additive conjoint structure for such conjoint analysis to be feasible. When this condition is satisfied, the decision maker can make a rational decision to a certain degree. The following describes the additive conjoint structure (Luce and Tukey 1964; Krantz et al. 1971).

Let X_1, X_2,... , X_q be the set of possible values of q attributes, and let the set of alternatives be the corresponding Cartesian product set, A $= X_1 \times X_2 \times \ldots . \times X_q$. A particular alternative x $\in A$ is given by $x = (x_1, x_2, \ldots, x_q)$, $y = (y_1, y_2, \ldots, y_q)$, where $x_1, x_2, \ldots, x_q, y_1, y_2, \ldots, y_q$ are the corresponding values of $X_1, X_2, \ldots, X_q$, respectively. If the following relation, $x \succsim y \Leftrightarrow u(x) \geq u(y)$, holds, the following additive utility functions represent the preference structure.

$$u(x) = \sum_{j=1}^{q} u_j\left(x_j\right)$$

$$u(y) = \sum_{j=1}^{q} u_j\left(y_j\right)$$

Krantz et al. (1971) pointed out that the additive utility function represented the preference structure if and only if the additive conjoint structure holds. For simplicity, we assume that two attributes determine the weak ordering, $\succsim$.

Let X_1, X_2, be the set of possible values of q attributes, and let the set of alternatives be the corresponding Cartesian product set, $A = X_1 \times X_2$. Let x_1, y_1, a_1 be three independent levels of X_1, and let x_2, y_2, a_2 be three independent levels of X_2. Krantz et al. (1971) stated that the additive conjoint structure, firstly, requires the he independence axiom, in which the ordinal relation between two levels of an attribute is independent of any and all levels of the other attribute. That is, the independence axiom holds if and only if, for x_1, $y_1 \in X_1$, $(x_1, x_2) \succsim (y_1, x_2)$ for some $x_2 \in X_2$ implies that $(x_1, y_2) \succsim (y_1, y_2)$ for every $y_2 \in X_2$; and, for x_2, $y_2 \in X_2$, $(x_1, x_2) \succsim (x_1, y_2)$ for some $x_1 \in X_1$ implies that $(y_1, x_2) \succsim (y_1, y_2)$ for every $y_1 \in X_1$. Krantz et al. (1971) also requires other five axioms, that is, a weak ordering, Thomsen condition, a restricted solvability, Archimedean property, and a property that each attribute is essential.

Thomsen condition satisfies, for any $x_1, y_1, a_1 \in X_1$, and $x_2, y_2, a_2 \in X_2$, $(x_1, a_2) \sim (a_1, y_2)$ and $(a_1, x_2) \sim (y_1, a_2)$ imply $(x_1, x_2) \succsim (y_1, y_2)$. A relation $\succsim$ *on* $X_1 \times X_2$ satisfies restricted solvability provided that: (1) whenever there exist $x_1, \bar{y}_1, \underline{y}_1 \in X_1$ and $x_2, \bar{y}_2, \underline{y}_2 \in X_2$ for which $(\bar{y}_1, y_2) \succsim (x_1, x_2) \succsim (\underline{y}_1, y_2)$, then there exists $y_1 \in X_1$ such that $(y_1, y_2) \sim (x_1, x_2)$; (2) a similar condition holds on the second attribute. The *Archimedean condition* is the property of continuity named after the ancient Greek mathematician Archimedes. The Archimedean condition states that any strictly bounded sequence on either attribute is finite (Krantz et al. 1971). This property concerning infinite sets for example are basically untestable through direct observation. The property that attribute X_1 is essential requires there exists x_1, $y_1 \in X_1$ and $p_2 \in X_2$ such that not $((x_1, p_2) \sim (y_1, p_2))$. The property of the essentiality requires similar condition holds for X_2.

Multi-attribute decision-making that satisfies this additive conjoint system guarantees the best decision to be made, which allows maximization of multi-attribute utility. Whereas the empirical testing of solvability and Archimedean conditions is nearly impossible, the conditions of single cancellation and double cancellation can be tested empirically. Decision-making that satisfies these conditions would be difficult even in the study of double-attribute cases, which implies the difficulty of the conditions established. In this sense, assuming an additive conjoint system in multi-attribute decision-making is empirically difficult, suggesting the difficulty also of best decision-making.

3 Theoretical Examination When Multi-Attribute Decision-Making Does Not Dissatisfy a Weak Order

Quantitative analysis is easy if multi-attribute decision-making can be expressed by an additive system of utility of attributes as assumed in conjoint analysis and the preference is a weak order. As explained below, however, multi-attribute decision-making does not always have a weak order structure.

First, we will consider preference based on the dominance principle presented below as an example of the property of connectivity of a weak order.

3.1 Preference Based on the Dominance Principle

Preference that "x is indifferent or preferred to y" as the overall preference only when the preference relation of all attributes is "x is indifferent or preferred to y." In other words, there is a type of preference based on the dominance principle that becomes $x \succsim y$ only when "$x \succsim_i y$ for all attributes i." The following theorem holds for the dominance principle based on the properties below.

1. *Connectivity*: Relation $(\forall\ x, y \in A)$, $x \succsim y$ or $y \succsim x$ holds for arbitrary elements x, y of the set of alternatives A that is expressed by multiple attributes. In addition, the preference relation $x \succsim_i$ for different values of a given Attribute i satisfies connectivity.
2. *Transitivity*: Relation $x \succsim z$ holds if $x \succsim y$, $y \succsim z$ for arbitrary elements x, y, z $(\forall\ x, y, z \in A)$ of the set of alternatives A expressed by multiple attributes. In addition, the preference relation $x \succsim_i$ for different values of a given attribute i satisfies transitivity.
3. *No limitation of space for multi-attribute decision making problems*: As long as connectivity and transitivity are satisfied, any type of preference is applicable to each attribute value in multi-attribute decision-making and any combination of preferences can be made among the multiple attributes.

Theorem of decision-making based on the dominance principle Preference based on the dominance principle under the above conditions (1), (2), and (3) above, i.e., the preference that becomes $x \succsim y$ only when "$x \succsim_i y$ for all attributes i" does not satisfy connectivity, is not a weak order, and involves no multi-attribute value function that expresses a preference relation.

Proof Consider a case of preference based on the dominance principle in which $x \succsim_k y$ holds for a given attribute k and y is weakly preferred to x for other attributes. In this case, connectivity cannot be established because $x \succsim y$ and $y \succsim x$ do not hold to begin with. The above theorem, therefore, holds true.

This result suggests that such decision-making based on the dominance principle assumed for a number of psychological models would be difficult to perform even for ordinary quantitative analysis, which differs from the so-called principle of utility maximization (Takemura 2011a, b).

3.2 Preference Based on the Principle of the Maximum Number of Dominant Attributes

Preference that "x is indifferent or preferred to y" as the overall preference only when the number of attributes whose preference relation is "x is indifferent or preferred to y" is large. The following theorem holds also for this type of reference.

Theorem of decision-making based on the principle of the maximum number of dominant attributes Preference based on the principle of the maximum number of dominant attributes under the conditions of (1) connectivity, (2) transitivity, and (3) no limitation of problem space is not a weak order and does not involve any multi-attribute value function that expresses a preference relation.

Proof If the number of attributes is an even number and the number of dominant attributes is the same, then none of them will be preferred. They will incomparable, which will not satisfy connectivity. If the number of attributes is an odd number, a cyclic order that does not satisfy transitivity can be made easily. The preference relation above, therefore, is not weak order, which does not involve any multi-attribute value function.

This consequently suggests that decision-making would be difficult with multiple attributes. Nevertheless, people must make decisions even under such conditions.

4 Theorem of Impossibility of Multi-Attribute Decision-Making

Decision-making for multiple purposes that considers pluralistic aspects such as monetary value, ethical value, and aesthetic value has a structure called "multi-attribute decision-making" in the decision theory. Multi-attribute decision-making considers a multi-attribute alternative x as an alternative expressed by q-dimensional attributes and the set of attributes $X_1, X_2, \ldots, X_q$ expressing various values as the elements of the subspace of the Cartesian product set. In other words,

$$x \in X_1 \times X_2 \times \ldots \times X_q.$$

Assuming a Cartesian product $X_k \times X_k$ for an arbitrary attribute k, we consider that the ordered pair of this element expresses the preference relation of the attribute value. If this preference relation is R_k, R_k is a subset of $X_k \times X_k$. It is natural to assume that R_k satisfies the properties of transitivity and connectivity for each attribute as the basis of rationality.

When $R^q = R_1 \times R_2 \times \ldots \times R_q$, we assume that the function to apply the preference relation R based on multi-attribute decision-making to the elements of R^q is called the multi-attribute value function. In other words, the multi-attribute value function U can be expressed as shown below.

$$U : R^q \rightarrow R.$$

It is also natural to assume a weak order for the multi-attribute value function. Although the definition of the multi-attribute value function presented up to this point does not contradict the definition of an ordinary multi-attribute utility theory, adding the following conditions would make it different from the system of an ordinary multi-attribute utility theory. Such conditions are added because it would presumably be more natural as a starting point of an argument to assume that, as described in the Introduction, the incomparability (difficulty of comparison) of various values and each value can only be judged in the sense of ordinal numbers.

In addition, as stated in the Introduction, one social choice theory that is used to consider decision-making of social groups is Arrow's general possibility theorem (Arrow 1951) of democratic group decision-making. The conditions presented by this theorem are of a multi-attribute decision-making problem and re-interpreting the results in the general possibility theorem of the following multi-attribute decision-making. This theorem indicates that composing a multi-attribute value function that satisfies all the conditions below is impossible, meaning that conditions to satisfy connectivity and transitivity, which are the conditions for rationality, and the following conditions presumably appropriate for rational decision-making do not hold simultaneously.

4.1 Conclusion of General Possibility Theorem of Multi-Attribute Decision-Making

We assume that the Cartesian product of Sets $X_1, X_2, \ldots, X_q$ of attributes that express various values is A, and its elements (ordered set expressing the q-term relations) such as x, y, and z are alternatives described by multiple attributes. If there are three or more multi-attribute alternatives described by two or more attributes, there is no multi-attribute value function that satisfies the following conditions, and satisfying these conditions simultaneously would be contradictory. The value function that satisfies the conditions (1), (2), (4), and (5) under this condition of three or more alternatives with two or more attributes is one-dimensional or expresses an imposed preference relation. In this case, being one-dimensional means that preference is expressed only by a single-attribute preference relation, and being imposed means that a preference relation is determined for a given pair of alternatives irrespective of the values of attributes.

1. *Connectivity*: Relation $(\forall x, y \in A), x \succsim y$ or $y \succsim x$ holds for arbitrary elements x, y of the set of alternatives A that is expressed by multiple attributes. Additionally, the preference relation $x \succsim i$ for different values of a given attribute i satisfies connectivity.
2. *Transitivity*: Relation $x \succsim z$ holds if $x \succsim y, y \succsim z$ for arbitrary elements x, y, z $(\forall x, y, z \in A)$ of the set of alternatives A expressed by multiple attributes. Additionally, the preference relation $x \succsim i$ for different values of a given attribute i satisfies transitivity.
3. *No limitation of space for multi attribute decision making problems*: As long as connectivity and transitivity are satisfied, a preference of any type is applicable to each attribute value in multi-attribute decision-making and any combination of preferences can be made among the multiple attributes.

4. *Monotonicity (Pareto principle)*: If the preference relation of all attributes is "x is preferred to y," the overall preference is also "x is preferred to y" (In other words, "$x \succsim_i y$ for all attributes i " results in $x \succsim y$).
5. *Independence of irrelevant alternatives (IIA property)*: Preference for alternatives x and y is determined only by ordering the attributes of these two alternatives. In other words, they are unaffected by the attribute preference for the other alternative z. (Therefore, finding out whether $x \succsim y$ holds only requires a profile describing either one or both of $x \succsim_i y$ and $y \succsim_i x$ hold for those specific x, y in all cases of attribute i).
6. *Multiple goals (multi-dimensionality)*: No preference is based only on a single attribute (an attribute that always makes the overall preference of x over y if the attribute prefers x to y) (In other words, there is no attribute i that makes "$x \succsim y$ if $x \succsim_i y$ for an arbitrary preference profile). This condition demands that people make decisions for multiple goals and never make decisions only on one dimension.

If Arrow's general possibility theorem (Arrow 1951) is interpreted in this manner, then when three or more alternatives exist, no multi-attribute value function satisfies all the conditions for the six axioms related to the overall preference of the decision-maker described above. In other words, the two conditions (preferences can be comparable and transitive) for the decision-maker to make rational selection of alternatives and the four conditions suggesting the rationality of multi-attribute decision-making do not hold true simultaneously, which implies that it is extremely difficult for the decision-maker to make the optimal and rational decision in multi-attribute decision-making, which suggests that even if we seek rational decision-making to achieve our welfare, it is likely impossible as long as we have multiple dimensions and multiple goals. Considering that imposed decision-making is not rational, a rational decision might require decision-making based on a one-dimensional attribute. Although this seems to be rather counter-intuitive, it might be the key to explaining why an individual seeking rationality intends to make a one-dimensional judgment.

5 Descriptive Analysis of Multi-Attribute Decision-Making and the Psychology and Inclination for One Dimension in Decision-Making

5.1 Scale of Decision Strategy and Multi-Attribute Decision-Making

Although the discussion presented above suggests the difficulty of making the best decision in multi-attribute decision-making, the following examines how people actually make their decisions. In our laboratory, we use the following scale of decision-making for decision strategy in multi-attribute decision-making (Ideno et al. 2012).

Various biases and heuristics have been reported in the context of behavioral decision theory; there has not been much progress in the examination of the causes. Accordingly, we have developed a scale by application of the anomalies in behavioral decision theory that have been described in earlier reports.

We conducted an internet survey. Survey participants were 1,600 individuals, including age cohorts of 200 each (100 men and 100 women consisting of 20 each in their 20s, 30s, 40s, 50s, and 60s) assigned to seven locations (Sapporo, Sendai, Tokyo metropolitan area, Nagoya, Osaka, Hiroshima and Matsuyama, and Fukuoka). The scale of decision-making used for this study was developed based on items extracted from various theories and phenomena related to decision-making. The following are some examples of such items. These include "the most preferred alternative would be the same in any combination of alternatives to compare" for transitivity, "there should be a combination of alternatives that could not be compared" for comparability, "would examine whether a fairly satisfactory alternative exists" for satisficing, and "Prefer to avoid a decision whose probability is not known" for tendency of uncertainty avoidance. Table 13.1 presents details.

We used the survey results to perform factor analysis for the decision-making scale using the principal factor method for factor extraction and Promax rotation for axis rotation. Table 13.2 presents the results, including the five factors—(1) careful consideration, (2) uncertainty preference, (3) uncertainty avoidance, (4) satisficing, and (5) consistency—that were extracted. The factors correlated mutually, and a correlation coefficient of 0.3 or above was observed between the careful consideration factor and three other factors (0.33 between careful decision-making factor and uncertainty factor, 0.33 between the careful decision-making factor and satisficing factor, and 0.39 between the careful decision-making factor and consistency factor). Furthermore, a factor correlation of 0.4 was found between the uncertainty avoidance factor and satisficing factor.

The results presented above have revealed the central role in decision strategy played by the careful consideration type of decision-making. The act of making the best decision is included in this careful consideration type of decision-making, demonstrating the high correlation between decision-making based on multiple attributes and attempts to make the best decision. Table 13.3 shows that the satisficing strategy positioned opposite of careful decision-making maintains a high correlation with careful decision-making and also that it has strong ties with uncertainty avoidance. This also supports the finding that a psychological connection exists between best decision making and the satisficing type of decision-making.

Table 13.1 Decision scale

Able to arrange any set of alternatives in the order of preference
Prefer to decide by considering only the most important characteristics
The most preferred alternative would be the same in any combination of alternatives to compare
Seek the optimal alternative
Should be able to compare any combination of alternatives
Carefully examine all alternatives
Look no further once a satisfactory alternative is found
Factors to focus change depending on the conditions for selection
Prefer to select a risky alternative
Prefer to decide by considering as many characteristics of alternatives as possible
Prefer to avoid an alternative for which the result or the probability of the result is known
Prefer to make a safe decision
Prefer to select an alternative for which the result and its probability is NOT clearly known
Prefer to avoid a decision for which the result and its probability are NOT known
Prefer to select an alternative for which the consequences are unknown
There should be a combination of alternatives that could NOT be compared.
Always consider whether there is the best alternative
Prefer an alternative that guarantees a profit even if the amount is relatively small
Prefer to select only those alternatives that are good in all aspects
The mode of making a decision changes when the issue becomes more complex
Prefer to avoid an alternative for which the result is predicable
Attempt to consider as many perspectives as possible rather than a single perspective
Tend to select an alternative if it is fairly satisfactory
Examine whether a fairly satisfactory alternative exists
Select an alternative that includes positive factors even if it also includes negative factors
Prefer to avoid a decision whose probability is NOT known

These were assessed using the following seven-subject method.

1. Not applicable at all
2. Not applicable
3. Not very applicable
4. Not sure
5. Somewhat applicable
6. Applicable
7. Very applicable

Source: Ideno et al. (2012)

Table 13.2 Results of factor analysis based on decision-making scale

Question	PA1	PA2	PA3	PA4	PA5	Factor
Always consider whether there is the best alternative	0.75	−0.09	0.00	−0.03	0.05	Careful decision-making
Seek the optimal alternative	0.71	−0.19	−0.03	0.02	0.17	
Attempt to consider as many perspectives as possible rather than a single perspective	0.67	0.08	−0.10	0.00	−0.01	

(continued)

Table 13.2 (continued)

Question	PA1	PA2	PA3	PA4	PA5	Factor
Prefer to decide by considering as many characteristics of alternatives as possible	0.65	0.07	0.12	−0.02	−0.09	
Carefully examine all alternatives	0.63	0.09	0.07	−0.12	0.07	
Factors to focus change depending on the conditions for selection	0.51	0.02	−0.01	0.17	−0.01	
There should be some combination of alternatives that could NOT be compared	0.48	0.05	−0.04	0.10	−0.15	
Prefer to select an alternative for which the consequences are unknown	−0.05	0.70	−0.06	0.00	−0.06	Uncertainty preference (Gambler)
Prefer to select an alternative for which the result and its probability are NOT clearly known	−0.10	0.63	−0.02	0.12	0.01	
Prefer to select a risky alternative	0.07	0.62	−0.03	−0.10	−0.10	
Prefer to avoid an alternative for which the result is predicable	−0.13	0.49	0.12	0.04	0.10	
Prefer to avoid a decision for which the probability is NOT known	−0.03	0.02	0.90	−0.02	−0.01	Uncertainty avoidance
Prefer to avoid a decision for which the result and its probability are NOT known	0.07	−0.11	0.80	0.04	−0.02	
Tend to select an alternative if it is fairly satisfactory	−0.06	0.04	0.00	0.83	−0.01	Satisfying
Examine whether a fairly astisfactory alternative exists	0.04	0.00	0.01	0.79	0.02	
The most preferred alternative would be the same in any combination of alternatives to compare	0.11	−0.16	−0.04	0.01	0.72	Consistency
Able to arrange any set of alternatives in the order of preference	0.15	0.17	0.00	−0.13	0.52	
Prefer to decide by considering only the most important characteristics	−0.02	0.09	0.03	0.13	0.51	
(Total contribution rate)	0.16	0.25	0.34	0.42	0.48	

Source: Ideno et al. (2012)

Table 13.3 Relations with other decision-making scales

		CRL	CRL	CRL	DM	DM	DM	DM	DM
		Goal-oriented	Situation-dependent	Simplifying	Careful	Uncertainty preference	Uncertainty avoidance	Satisfying	Consistency
CRL	Goal-oriented	1.00							
CRL	Situation-dependent	0.55	1.00						
CRL	Simplifying	−0.07	0.13	1.00					
DM	Careful	0.67	0.57	−0.02	1.00				
DM	Uncertainty preference	−0.06	−0.10	0.25	−0.10	1.00			
DM	Uncertainty avoidance	0.22	0.23	−0.03	0.29	−0.16	1.00		
DM	Satisfying	0.12	0.31	0.12	0.27	0.06	0.31	1.00	
DM	Consistency	0.33	0.34	0.20	0.37	0.15	0.16	0.21	1.00

Source: Ideno et al. (2012)

As demonstrated up to this point, seeking the best decision is related to considering multiple attributes in decision-making. As suggested by the earlier theorem of impossibility, however, multi-attribute decision-making make the best decision-making extremely difficult. Then, how do people make their decisions in real-life decision-making?

After given a lecture on the difficulty of decision strategy and multi-attribute decision-making in the class on decision theory in the Department of Psychology at Waseda University and the University of Tokyo, 38 graduate and undergraduate students aged between 20 and 32 were asked to discuss how decisions should be made when the best decision cannot be made in multi-attribute decision-making. After the discussion, they were asked to select the most preferable alternative from (1) making the ultimate decision by abstracting the multiple attributes and using only one-dimensional attributes (change to one dimension), (2) compromising the best decision to make satisficing decision-making (satisficing), and (3) distorting their understanding to believe that the best decision is made (psychological dominance structuring). As a consequence, alternative 1 was selected by 11 students, alternative 2 was chosen by 16 students, and alternative 3 was chosen by 11 students. Therefore, almost one-third of the respondents would attempt to make the attributes one-dimensional in multi-attribute decision-making. Furthermore, many of them attempted to make the attributes one-dimensional based on monetary values and benefits. It is also interesting that nearly one-third of the respondents would adhere to the best decision even by distorting their understanding. More than a half of them would go through psychological manipulation to turn the attributes to one-dimensional or distort their understanding if the best decision cannot be made in multi-attribute decision-making.

5.2 *Mental Ruler Model in Multi-Attribute Decision Making*

The findings presented above suggest that rational decision-making is extremely difficult in general considering the multiple attributes involved and that decision-making based on one-dimensional attribute satisfies rationality. This analysis, however, is still based on an ideological perspective. In view of behavioral decision theory that describes the actual decision-making, how do people tend to make decisions in their daily life?

Based on the conclusions presented to this point and through decision-making studies and behavior observation, the author proposes a psychological model called the "mental ruler model," assuming that people tend to make a decision based on a one-dimensional attribute in multi-attribute decision-making (Takemura 1998, 2001). This model includes the assumption that, for instance, people tend to make the overall evaluation of universities based only on their standard scores merely representing the difficulty of their entrance examinations, or people's performance is assessed one-dimensionally based only on their sales figures, the number of papers written, or some criterion such as an impact factor.

5.2.1 Qualitative Description of "Mental Ruler"

Basic Hypothesis of the Model and Basic Property of Mental Ruler

From this point, the "mental ruler" model is presented to solve the problems described previously and to develop basic ideas of a decision frame model (Tversky and Kahneman 1981) and psychological purse model (Kojima 1959, 1994).

The basic hypothesis of this model postulates that people make decisions as if they have a ruler. In everyday life, it is often said metaphorically that every person uses a ruler with a different value to make decisions. Although a metaphor of this kind is valid only in our daily conversations, if we consider this metaphor thoroughly and scientifically, it is more useful than we might think to explain contingent decision-making. Objects of the mental ruler are divisible into gain and loss areas just as the decision frame model, but phenomena that cannot be classified into gain and loss areas can be included, such as the judgment on personal impressions such as those related to generosity or calmness, or a judgment related to probability.

Let us first consider the basic meaning of "ruler." A ruler is used to measure "length". The reason people use a ruler is, of course, that merely looking at an object is not good in judgment on length because it causes unevenness or distortion. Psychologically, people cannot judge with confidence without using a ruler. We use a ruler as a standard for judgment. A physical ruler enables us to judge length with certainty and relief. What do we do if we have no physical ruler? I assume that people construct a ruler internally in their mind in a sense for such a situation. This can be regarded as a creative process in recognition of decision-making problem.

Next, one can consider the mental ruler characteristics further, and capture and discuss the characteristics of decision-making metaphorically.

1. Basic Property 1: The ruler has graduation.
   The author assumes that people make a decision based on the graduation of the mental ruler, which can be fine or rough, just as units of millimeter or centimeter on the graduation of a physical ruler. For example, let us consider a judgment of price. With fine graduation, consumers must be sensitive to a difference by even 1 cent. In contrast, with rough graduation, they can be insensitive to a difference by several units of $100. Such a difference of sensibility about prices can be described using the fineness or roughness of the graduation of the mental ruler. As described later, we can imagine the roughness of the graduation of the ruler might change for the same person depending on the situation.
2. Basic Property 2: The ruler length is bounded (boundedness).
   This property seems quite basic, but the metaphor denotes a great deal. For example, for judgment related to price, we cannot judge easily whether the price of an object exceeds the length of the mental ruler greatly in both directions, i.e. when the price is extremely high or extremely low. Consumers might joint several rulers when the ruler is too short, but the elicited judgment probably varies widely.

3. Basic Property 3: The ruler is one-dimensional.
   A physical ruler measures a one-dimensional property called length. Even though people make a judgment founded on multidimensional information, it is quite possible that they finalize the judgment one-dimensionally. In Japan, many people think that education based on the standardized value of test scores is not good, but simultaneously they tend to be concerned about the standardized value of test scores very much. People like to check rankings of various kinds, such as a "best seller" ranking at a shop. These tendencies seem to indicate an important facet of human nature: one-dimensional judgment.

Basic Function of Mental Ruler

Based on the basic properties of the mental ruler described above, some theoretical predictions about its basic functions are presented below.

1. Basic Function 1: People construct an appropriate mental ruler depending on the situation.
   People construct a mental ruler with appropriate graduation and of appropriate size, depending on the situation. People do this so naturally that they usually do not perceive it themselves. This phenomenon, however, can often be perceived if we compare purchasing situations. For example, in Japan, if a person thinks of purchasing a new car, a consumer constructs a mental ruler with graduation of $100 unit when negotiating with a car dealer about the price or optional equipment because a brand-new car often costs more than $10,000. In such a situation, a price differential of several $1 is treated as an error, and is seldom examined. The same consumer, however, goes to a supermarket after the car dealer and can be satisfied with the price of a package of 10 eggs that is lower than usual by 20 cents, or be disappointed by a price that is higher than usual by 30 cents and might not buy the eggs. A person concerned about a price of 10-cents units to make a judgment or a decision in this situation. Similarly, we can presume that people specifically examine the ongoing situation and construct the situation subjectively, and construct a mental ruler upon the situation.
2. Basic Function 2: Reference points or endpoints of the ruler are applied differently depending on the situation.
   For example, in judging on price, a reference point changes according to the object group that is compared. Either a price that is lower than that at another shop or than a prior price makes the reference point of the ruler move to a different position. A judgment of the price or the decision on the purchase might be changed. The ruler endpoints are also assumed to change according to the situation such as a comparison of groups of objects.
3. Basic Function 3: Graduation of the ruler becomes particularly finer around the reference point and the endpoints (nonlinearity of the ruler).
   This property does not apply to a physical ruler. For instance, a consumer who is trying to buy an article for the budget of $100 becomes more sensitive to the

difference between $95 and $100, than that between $50 and $55. It becomes extremely difficult to evaluate if the comparing prices exceed the endpoints. For instance, if the budget is $100, then the consumer becomes insensitive to the difference between $150 and $155. The evaluation becomes unstable.

4. Basic Function 4: More knowledge or more involvement creates finer graduation of the ruler.
   If a consumer has much knowledge related to an article, alternatively if a consumer is involved in an article to a great degree, the graduation of the ruler becomes finer. Then the consumer becomes sensitive to small differences, which engenders classification of similar articles very precisely. Therefore, it happens that the consumer tends to buy the article at a higher price if and only if its quality is only a little better than that of the others.
5. Basic Function 5: Even if information is given multidimensionally, a one-dimensional judgment is elicited using the mental ruler.
   This not only denotes that people merely simplify the problem while avoiding the data processing load. Consumers might construct another ruler to cope with the situation as a kind of creative process in the recognition of decision-making problem. For instance, by reading fashion magazines or through repeated shopping experiences, consumers construct a ruler such as "good taste" based on the complicated information about clothes to make a purchase decision. The mental ruler in this case is also fundamentally one-dimensional.
6. Basic Function 6: It is difficult to compare different mental rulers.
   It is presumably difficult for consumers to compare and to evaluate various mental rulers themselves they have constructed mentally, depending on the situation. Such contradictory judgments or decisions among situations such as examples of a car purchase and an egg purchase cannot be perceived by the consumers themselves, which is true because people usually specifically examine the situation, construct the situation subjectively, and construct a mental ruler on the situation. It therefore becomes difficult to construct more than two rulers for one situation from the cognitive load perspective. People sometimes use a different ruler for the same value from an economic rationality perspective, or use the same ruler for situations in which they should use different rulers.

Compatibility of Stimulus–Response Structures as a Mental Ruler Construction Principle

Lastly, I discuss the mental ruler construction principle.

I presume that the compatibility of stimulus–response structures plays an important role in constructing the mental ruler. The compatibility of stimulus–response structures denotes compatibility between structural characteristics of the input mode and response mode (Selart 1997). The efficiency of the information process in a judgment or a decision increases if they match or correspond well. I assume that a consumer constructs a mental ruler as an input mode corresponding to a given response mode. For instance, the purchase choice situation "to buy or not to buy" has a two-valued response mode; the consumer constructs a two-valued mental ruler, "good or bad." However, if a consumer is asked to evaluate an article by ranking or by points, then the consumer constructs a mental ruler of multiple values.

A consumer has difficulty in judgment if the stimulus–response structure does not correspond well. For instance, if a mental ruler has already been constructed, then a consumer cannot judge precisely if the ruler only has rough graduation, and vice versa.

From this compatibility of a stimulus–response structure perspective, too, the reason why the mental ruler is one-dimensional might be explained. The environment's structure requires a one-dimensional response mode of the judgment or the decision. Therefore, the mental ruler becomes one-dimensional. In addition, one can also assume that we often use linguistic terms of dual values such as "good or not good" to evaluate merchandise and so forth, which is true because decisions are constructed in dual-value response modes such as "to buy or not to buy."

5.2.2 Mental Ruler Explanation Using Set Theory and Its Mathematical Description

For simplification, an explanation is presented here using set theory for the mental ruler and its partial mathematical description. Below, I elucidate the qualitative and metaphorical description that was mentioned earlier by adding the structure. Therefore, no qualitative or metaphorical description is perfectly retrieved. Nevertheless, to create a psychometric model or to conduct various quantitative experiments, we must undertake formulation to some degree. To that end, the following is attempted.

Definition of the Situation

Let X' denote the whole situation to be discussed. Actually, X' is generally regarded as a finite set. Let ($S' \subset X'$), which is a subset of X', denote the focused situation. For instance, presuming that X' denotes the purchasing situation in a supermarket, and presuming that S can be a situation in which one must decide whether to buy some cola or not, or whether to buy a set of five notebooks or not, etc., then problem here is the focused situation that is determined cognitively by the decision-maker. In fact, although it is more natural to presume that a situation S' denotes a subset of X' for the Cartesian product ($S' \subset X' \times X' \times \ldots \times X'$) because situations are often a set of relation in a situation, one can presume a state S a subset of X' for simplicity. An important point here is that S' is subject to how the decision-maker pays attention: S' will have a different element if the same person specifically examines another side of the same situation, according to one's mood. Nevertheless, the hypothesis here is that S' is a commonly subjective situation that can be recognized by other people, too. S', which can be denoted extensively, is a set of events which exist over an individual's subject. For example, whether an article of \$10 sells for \$2 off or for 20 % off is the same situation, as long as the meaning of the event is stated denotatively.

Definition of Subjective Situation

Next, we assess the subjective situation. Let the limited set X denote the entire subjective situation, whereas S represents the subjective situation surrounding the decision-maker. Therefore, X', the set of the whole situation, corresponds to X, and S', the subjective situation, corresponds to S. One element in a situation, however, can have more than two elements in a subjective situation because the compatibility of X' and X, and S' and S are a many-to-one mapping (univalent correspondence) from the subjective situations to the objective situations. For instance, although both descriptions—"\$2 off" and "20 % off"—for an article of \$10 represent the same choice as long as they denote an event extensionally, they can be different elements in a subjective situation. Furthermore, even if in the same situation S', plural subjective situations are regarded as existing, such as the subjective situation $S_1, S_2, \ldots, S_n$ which is subject to the mode of the mental structure. Consequently, mapping f to the situation S' is regarded as being subject to the mode of the mental structure on the decision-making problem and can exist like $f_1, f_2, \ldots, f_n$. The set of these functions F ($f_1, f_2, \ldots, f_n \in F$) is regarded as constrained according to the cognitive ability of human beings [e.g., (Holyoak and Thagard 1995)]. Finally, the mapping from the subjective situation to the objective situation, f, is not onto mapping generally. Therefore, elements of the subjective situation do not necessarily cover the elements of a situation but can be omitted partially, which can be considered because of the cognitive constraints of the decision-maker's attention, memory capacity or searching ability. Presumably, mapping has the direction to promote the stimulus–response compatibility stated earlier, or to create the dominance structure (Montgomery 1983, 1993) in a decision-making problem.

Structure of a Mental Ruler

Therefore, the mental ruler can be defined. The mental ruler approximately differs with positive and negative areas, just like the value function in the prospect theory. The greater the number, the better it is in one case and the worse it is in another. Nevertheless, as stated earlier, the objects of the mental ruler model also include rather neutral ones such as probability judgment, not only the gain and loss areas. For simplification, however, this matter is discussed only in the positive area. Moreover, the author first discusses the mental ruler model for a case in which the evaluation object as an element of subjective situation S can be described objectively using an additive measurement such as price, length, or size. Then, the mental ruler can be described as a set function from the subsets of the subjective situation S to one-dimensional real number space R.

First consider a case in which an element x of the subjective situation, S can be described objectively as an additive function to price, length, proportion, probability, and so forth, as $m(x) \in R$. Consider a function m from S to one-dimensional real number space R, m: $S \rightarrow R$. For instance, let $m(x)$ denote the discount rate m for an article x. Moreover, consider the mental ruler using the function v from one-dimensional real number space R, which is mapped by m, to one-dimensional real number space R, which describes the evaluation value v: $R \rightarrow R$. Here, v has the following property.

$$m(x) = 0 \rightarrow v(m(x)) = 0 \tag{13.1}$$

$$x^* = \underset{x \in S}{\text{argmax}}\, m(x) \rightarrow v(m(x^*)) = k, \text{where } k \text{ is a constant.} \tag{13.2}$$

$$m(x) \geq m(y) \rightarrow v(m(x)) \geq v(m(y)) \tag{13.3}$$

Formulas (13.1) and (13.2) denote the boundedness of the mental ruler. For example, the evaluation for the relative income of \$0 is 0, where the evaluation for the evaluation object that has the most value in the subjective situation is a real number k. Here, x^* denotes x, which maximizes $m(x)$. For instance, when the upper limit of a relative income is regarded as \$10,000, then the alternative, which gives \$10,000 is x^*. Alternatively, considering the evaluation for a price using the mental ruler, if the upper limit of the budget is \$100, then the article equivalent to the \$100 is equivalent to x^*. Formula (13.3) describes the monotonicity of the mental ruler, which suggests that evaluation using the mental ruler does not exceed k in the subjective situation S. Moreover, if the mental ruler is unique with regard to the positively proportional transformation (the similarity transformation), by an adequate scale transformation, then

$$v(m(x^*)) = 1, \text{ where } x^* = \underset{x \in S}{\text{argmax}}\, m(x) \tag{13.4}$$

As stated earlier, x^* denotes x which maximizes $m(x)$, where x^* always denotes the same quantity in the following discussion. Additionally, for simplification, the evaluation function of the mental ruler is presumed to hold always for Formula (13.4) in the discussion below.

Subadditivity of the Mental Ruler and Its Mathematical Description

Although the mental ruler has monotonicity of Formula (13.3), it has no additivity such as the following.

$$v(m(x) + m(y)) = v(m(x)) + v(m(x)) \tag{13.5}$$

The mental ruler is regarded as holding subadditivity of the following two kinds (Tversky and Fox 1995; Tversky and Wakker 1995).

1. Lower Subadditivity $v(m(x)) \geq v(m(x) + m(y)) - v(m(y))$, where
$$m(x) + m(y) \geq 1 - \varepsilon, \varepsilon \geq 0, \tag{13.6}$$

Formula (13.6) shows that the evaluation function becomes concave downward when $m(x)$ is low. This property is the same as that of the weighting function for lower probability in the prospect theory; it is also the same as that of diminishing marginal utility in the utility theory.

2. Upper Subadditivity $v(m(x^*)) - v(m(x^*) - m(x)) \geq v(m(x) + m(y)) - v(m(y))$,
$$\text{where } m(x) \geq \varepsilon', \varepsilon' \geq 0, \tag{13.7}$$

This property is related to an event for which the evaluation function of the mental ruler becomes convex downward when $m(x)$ is high. This is the same as the property of the certainty effect indicating that the weighting of probability 1 is much greater than the probability less than 1, as explained using the prospect theory.

The mental ruler model, however, forecasts that this property holds not only with the weighting probability but also with the values of the outcomes. This forecast is completely contrary to the property of the diminishing marginal utility in the utility theory or in the prospect theory. In the utility theory or the prospect theory, a function that is concave downward is always assumed, although the mental ruler model includes the assumption that a function exists that is convex downward around the upper bound. For example, when negotiating on a discount for the price, the sensibility rises around the target. Let $m(x^*)$ denote the targeted gain for the negotiation. The function becomes convex downward around the targeted price, but it becomes concave downward around the zero gain, where $m(x) = 0$. The function which holds the property of Formulas (13.6) and (13.7) is an S-shaped function. An S-shaped mathematical description that has such a property is

$$v(m(x)) = \exp(-(-\ln(m(x)/m(x^*)))^{\gamma}) \tag{13.8}$$

Prelec (1998) originally used this function as a weighting function for probability (see Fig. 13.1). Here, $m(x)/m(x^*)$ takes interval [0, 1]; its price interval is also [0, 1]. Then the fixed point becomes $1/e \approx 0.36$ irrespective of the value of γ (Wu and Gonzalez 1996).

Another such function is:

$$v(m(x)) = \frac{(m(x)/m(x^*))^{\gamma}}{((m(x)/m(x^*))^{\gamma} + (1 - m(x)/m(x^*))^{\gamma})^{\lambda}} \tag{13.9}$$

Here, if $\lambda = 1$, then it is the same as probability weighting function of Karmakar (1978) (see Fig. 13.2); if $\lambda = 1/\gamma$, then it is the same as the probability weighting function presented by Tversky and Kahneman (1992).

Wu and Gonzalez (1996) conducted a psychological experiment to investigate the weighting function to probability, applied functions of many kinds, and proved that the function by Tversky and Kahneman (1992) showed high applicability, as did the function of Prelec (1998). They also proved that although the weighting function is concave downward around the probability 0.40, it becomes convex downward if it exceeds about 0.40. Although the weighting function they obtained is expressed only against the probability, it can be assumed that the expertise of the same kind is obtainable for the value such as money, from the mental ruler perspective. Additionally, it is already proved that the evaluation function of the number of surviving lives is S-shaped, as shown in the evaluation experiment later.

Moreover, the following function has been proposed neither in the research on the existing probability weighting function nor in the research on the prospect theory, but is now being proposed in Takemura (1998), and is assumed to be an evaluation function that satisfies upper subadditivity and lower subadditivity, as

$$\begin{aligned} &v(m(x)) = w_1(m(x)/m(x^*))^{\alpha} + w_2\Big(1-(1-m(x)/m(x^*))^{\beta}\Big), \\ &\text{where}\, w_1 \geq 0, \quad w_2 \geq 0, \quad w_1 + w_2 = 1, \end{aligned} \tag{13.10}$$

Therein, if $\lambda = 1$, then it is the same as probability weighting function of Karmakar (1978) (see Fig. 13.2); if $\lambda = 1/\gamma$, then it is the same as the probability weighting function of Tversky and Kahneman (1992).

This section briefly described that the mathematical system underlying utility theory has difficulty in completely explaining contingent decision-making. Subsequently, the author introduced the qualitative decision frame model (Tversky and Kahneman 1981) and the psychological purse model (Kojima 1959, 1994) of contingent decision-making. Finally, this chapter introduced the "mental ruler" model to explain contingent decision-making qualitatively. The basic hypotheses were considered along with the basic function of the mental ruler and the basic structural theories. The concept of the model is that people judge or make decisions using a mental ruler that is constructed for subjective situation, which was structured as a support so that a one-dimensional mental ruler can be easily made on it. The main characteristic of the model is that—contrary to the recent utility theory or prospect theory—it treats utility or value and subjective probability as fundamentally the same evaluation function. The author also described the instability of the judgment in the area beyond the length of the mental ruler: not as in previous theories. The mathematical model was presented for its more rigorous formulation in the future.

This explanation presented in this chapter specifically described the one-dimensionality of the evaluation in judgment and decision-making. The discussion, however, has restrictions. In some cases, people evaluate multidimensional attributes and evaluate information multidimensionally. For example, cases are often observed in which people make a judgment or a decision while consciously considering multidimensional information as assumed in multi-attribute attitude theory or multi-attribute decision-making theory. It will be necessary in the future to clarify situations in which one-dimensional evaluation, as assumed in the mental ruler model, occurs easily, and situations in which evaluation considering multidimensional information is done more often.

6 Focusing Attention on the Causes of One-Dimensional Decision-Making and "Good Decision-Making"

Apart from Takemura et al., Gigerenzer and his group have been actively claiming in recent years that people make decisions based on one-dimensional attributes. They are proposing the concept of fast and frugal heuristics and developing their studies of judgment and decision making. Initially, Gigerenzer and Goldstein (1996) used a computer simulation to illustrate that when selecting a city that is more populated than the other from two alternatives, a decision made using recognition heuristics, based only on whether the respondents knew the cities or not, would still be rational. They demonstrated that a property not directly related to the issue of comparative judgment such as the ease of recognition might be an important factor in the decision (Gigerenzer and Goldstein 1996). The series of studies begun with this were not originally research on decision making, but were attempts to present a counterexample to conventional research on judgment biases that availability heuristics relying on memory would engender irrational judgment. In 2006, however, by expanding the idea of fast and frugal heuristics to decision-making and by developing the concept of priority heuristics (Brandstätter et al. 2006), they demonstrated that various decision-making phenomena can be explained solely based on the assumption that decisions were made easily for only a single reason in most cases. They questioned the basic assumption of decision theory made in expected utility theory and prospect theory that people made decisions by combining the utility of the result, value, and probability. This aspect is shared by the mental ruler model. It similar to the assumption of the simple, one-dimensional process of decision-making based on the attention mechanism of the contingent focus model described next.

Although one-dimensional decision-making might lead the process in a rational form, it is exposed to some risk. In the case of developing the reconstruction policy after the Great East Japan Earthquake, for example, determining a policy based only on the economic efficiency or on safety without considering the economic aspect would not necessarily be "good" decision-making. In fact, many people tend to make assessments based on limited attributes and tend to draw conclusions even for extremely important decision-making, which should be given a warning and remedied in some cases.

Why do people often make decisions based on one-dimensional attributes? One might explain that one-dimensional assessment makes it easier to find an optimal alternative that satisfies various conditions for rationality and gives people psychological contentment. In other words, this claims that people distort objective recognition and made decisions without considering information from other attributes. The author believes that explaining such a phenomenon, including such psychologically defensive reasons, involves the psychological property called

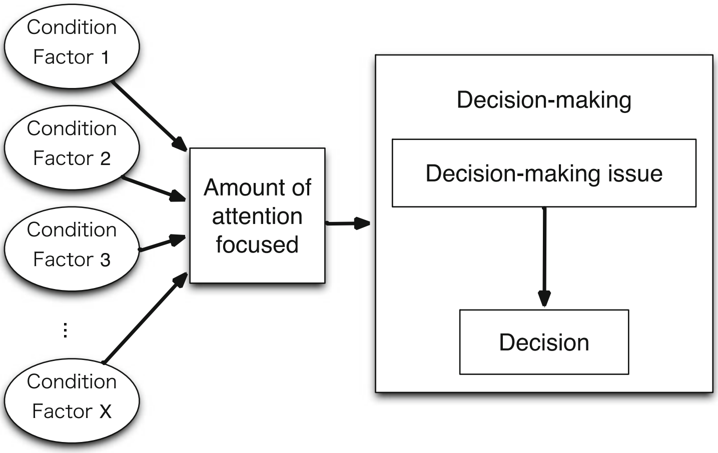

Fig. 13.2 Generalization of the contingent focus model

focus of attention. Focus of attention means that people's attention is drawn to specific attributes because of linguistic messages, image expressions, etc., and that decisions are often made based on the assessment of such attributes attracting the attention. Why does people's decision-making process depend on the situation or path? One reason might be that the range of our attention is limited and that decision-making is led by our attention. Based on that idea, we have developed a decision-making model called the contingent focus model (Takemura 1994), described in Chap. 10, to explain and forecast decision-making. When this model is generalized, the amount of attention to focus actually changes depending on the condition factors, as illustrated in Fig. 13.2, which causes the weight of decision-making attributes to change. Because decisions are made based on the changes, the decision-making process is expected to depend on the situation. Path dependency is explainable by the mode of making decisions that does not incorporate information related to all alternatives because of such a focus of attention and limitation of the attention range (Takemura 1996).

Those points explained above have been clarified to some degree by experimentation. We use the method of monitoring information acquisition with an eye movement measurement system to examine decision-making processes. Eye-movement-measuring devices include the contact type shown in Fig. 13.3 and the non-contact type in Fig. 13.4, which we use according to the research purposes. The contact type allows the test subject to move freely, which has the benefit of measuring the actual eye movement while making, for instance, a purchase decision. However, the range of vision shifts as the test subject moves, which makes data collection more difficult. We analyzed the decision-making process to seek the number of times the test subject focused the users' attention at each stage of decision-making, the average span of attention, the number of alternatives to which attention is devoted, and to the share of attention paid to the alternative that was eventually chosen. Our findings suggest, as shown in Fig. 13.5, that rather

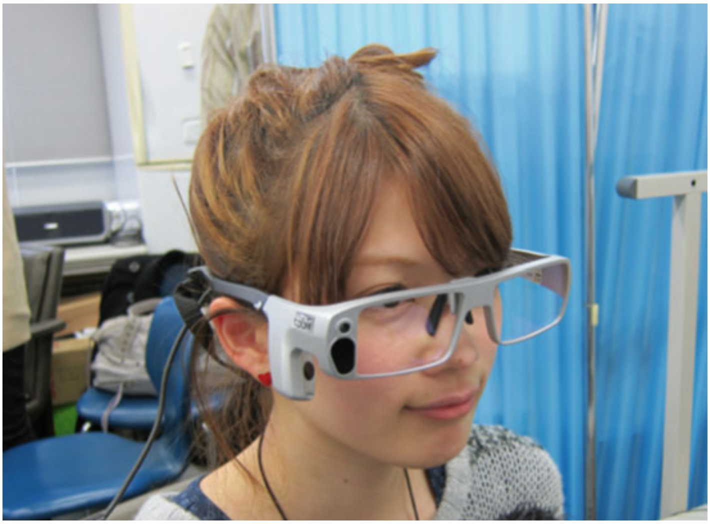

Fig. 13.3 Contact type of eye movement measurement system (Tobii Glass Eye Tracker, product of Tobii). *Note*: A camera to record the visual range of the test subject is amounted on the left side of the glasses. *Source*: Okubo and Takemura (2011)

Fig. 13.4 Non-contact type of eye movement measurement system. *Note*: Eyelink 1000 Remote, product of SR Research. *Source*: Okubo and Takemura (2011)

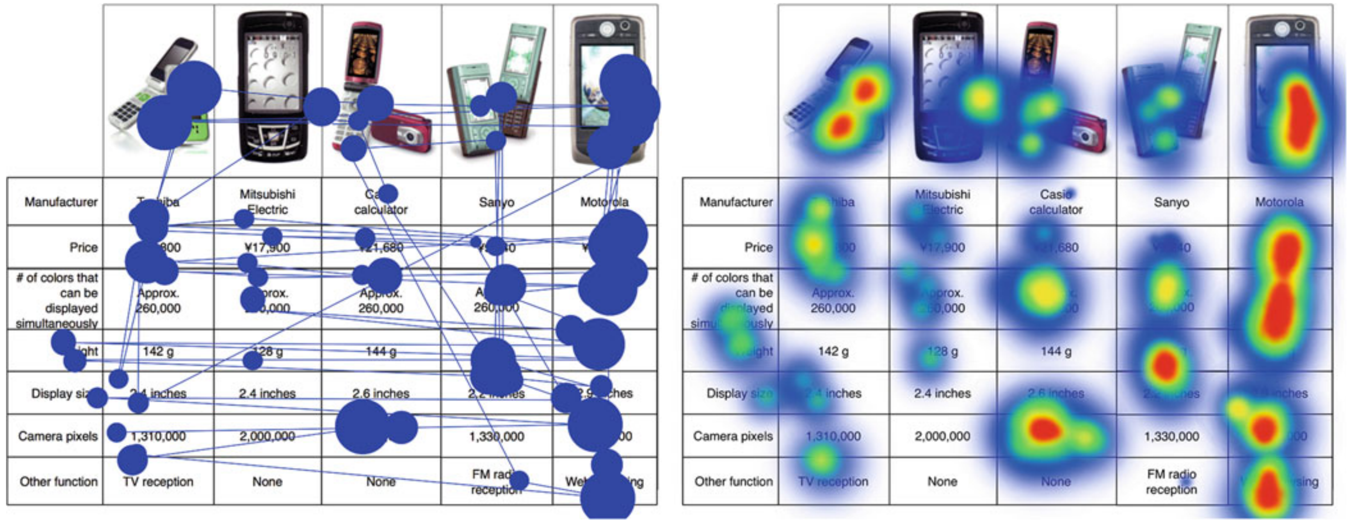

Fig. 13.5 Example of visualization of eyesight data on cell phone specifications table measured using an eye movement measurement system. *Note*: Figure on the right presents sections that attracted greater attention in order of red, yellow, green, and blue, similarly to thermography. *Source*: Okubo and Takemura (2011)

than equally observing all information available, people tend to make decisions based on considerably asymmetric information. This might be the reason why the process of people's decision-making tends to be situation-dependent and path-dependent (Fujii and Takemura 2001a, b, 2003, 2014; Takemura 1994).

The author and others generalized the contingent focus model described in Chap. 10 and established the function that maintains the following multi-attribute preference relations as the expression of multi-attribute utility function (Takemura et al. 2004). If α_{xk} is the parameter of focus of attention to the kth attribute of Alternative x, then only the parameter of attribute k is considerably large, which indicates that one-dimensional decision-making is taking place. Taking the logarithm of both sides while assuming that U is a positive value illustrates clearly that the relative sizes of the focus of attention has a linear effect on the logarithmic utility.

$$x \gtrsim y \Leftrightarrow U(x) \geq U(y), \text{ where}$$

$$U(x) = \prod_{k=1}^{q} u(x_k)^{\alpha_{xk}}$$

$$U(y) = \prod_{k=1}^{q} u(y_k)^{\alpha_{xk}}$$

The author and others are proposing a pshological model, designated as the contingent focus model, to explain decision-making based on such a focus of attention. This model explains that, in decision-making for risk taking, for example, the risk attitude changes depending on the level of attention focused on the result and probability. The author and others have revealed from psychological experiments of decision-making that results of decision-making that differ from those of prospect theory can be derived by manipulating attention to attribute information.

The contingent focus model suggests that preference changes by calling people's attention to other attributes. Based on such a perspective, Fujii et al. (2002) and Takemura and Fujii (2014) demonstrated that in decision-making not only for gambling tasks, but in that related to social dilemmas such as traffic issues, people's cooperation can be encouraged to a degree and some decisions to solve social dilemmas can be made by promoting attention to specific attributes with a payoff matrix. The fact that decision-making can be changed by attention implies that "good decision-making" might be possible by changing how information is presented and what information is emphasized. Calling people's attention to other attributes might cause confusion or reduce confidence in rational decision-making. However, it is important for the performance of pluralistic decision-making.

7 Towards Good Decision Making

Although Aristotle developed an argument on good decision-making on the assumption that the highest good existed, does one-dimensional value really exist? In the proof of Aristotle, something called welfare purportedly represented the highest good. However, he might have suggested welfare to be not only pleasure or not only virtue but something more pluralistic. According to Isaiah Berlin, however, the first one to argue the plurality of value in Western thought was Niccolo Machiavelli. Berlin was the one to explain it more clearly. Plurality of value means, for example, that when respect for human life is extremely important, freedom is also important. These two values are both important in an absolute sense but only one of them can be maintained under certain circumstances. A good example of an extreme situation is a dilemma in life, many of which are presented in literature as well. In reality, people argued at the time of the Great East Japan Earthquake whether to issue a large amount of government bonds to fund reconstruction in view of respect for human life and welfare or whether to prevent any issuance of a large amount of government bonds in view of economic stability of the country. This also demonstrates that coexistence of at least two values can present difficult circumstances. In this case, it is understandable that the two values are both important in an absolute sense and that both are reasonably important.

Currently, there might be three levels to the argument of plurality of value (Crowder 1994). The first is to perceive the plurality of value as a fact. The second is to require the plurality of value normatively. The last is a meta-ethical argument. Among these, the strongest argument for the value of plurality is the third one. The strong stance of plurality of value at the meta-ethical level is represented by Berlin's succession or interpretation of intellectual inheritance (Berlin 1969, 1990).

Berlin described the diversity of value, impossible coexistence and conflict of different values, and the incommensurability of value. These three elements constitute the core of today's value pluralism. Although people who argue the plurality of value generally approve these three aspects, their opinions are divided particularly in connection with the concept of incommensurability of value. George Crowder organized the incommensurability of value also into three stances (Crowder 2002).

The first is to interpret the incommensurability as an incomparability of value, which is the strongest interpretation of incommensurability. The second is to interpret incommensurability as the immeasurability of value, constituting the weakest interpretation of incommensurability. In this interpretation, only the returning of different values to "utility" just as in utilitarianism and quantitatively comparing them is rejected. Ordinal ordering of value, however, is allowed. The last is, in effect, a stance between these two, which is interpreted as the unrankability (impossibility of ranking) of value. This is the stance of Crowder himself. Crowder explained his value pluralism based on the following four aspects.

1. Existence of universal value: Certain values are universal and objective.
2. Plurality: Many values have crucial importance to people's prosperity.

3. Incommensurability: Values cannot be ranked in a manner that is irrelevant to the context or in an abstract manner.
4. Conflict of values: Conflict of values is not accidental but is unavoidable in the human world [(Crowder 2002), pp. 45–46].

In this chapter, too, we consider the stance of value pluralism argued by Crowder. The following discusses what good decision-making would be like when viewed from such a perspective. In other words, a "good decision" is regarded as pluralistic and to satisfy at least pluralistic standards such as formal rationality, welfare, justice, beauty, and virtue.

Decision-making from a pluralistic perspective is psychologically difficult, as discussed in this chapter, and it is also difficult in view of form. According to the author, however, psychological ease or rationality from a formal perspective does not necessarily engender a "good decision." In an extreme case, decision-making based on one-dimensional attributes satisfies rationality and facilitates the best decision-making. Although one-dimensional decision-making is easy for many people to perform, it is not necessarily preferable considering the plurality of value. In modern society, serious conditions arise in various situations, information to be processed comes in large amounts, and decision-makers carry a heavy psychological burden, which might increasingly drive people to resort to psychologically easy and formally rational decision-making. People are seemingly inclined towards the selection of decision-making falling into excessive formalism, for example, by overly applying legal compliance and accountability or demanding procedural rationality. In terms of value standards, many people seem to be attempting to make decisions only in view of formal procedures and fairness. Takahashi et al. (2010) conducted a questionnaire survey and suggested that the psychological tendency to pursue such formalism derived not from a moral sense, but from the intention of avoiding responsibility. Individuals showing a strong tendency to act this way were actually less altruistic than others. In addition, past studies in social psychology reveal that people seeking rationality tend to be rather susceptible to depression and a low level of subjective welfare (Schwartz et al. 2002). Considering these aspects, in the author's opinion, purposely placing oneself in the multidimensional confusion to consider multidimensional decision-making while recognizing rationality as an important standard might lead to "good decision-making."

Multidimensional thinking might cause information overload and might engender more confusion. People are therefore inclined towards one-dimensional thinking. However, what could be done to prevent such a tendency? One strategy is to narrow down the attributes to a few important ones or to abstract multiple attributes appropriately. In this way, the information load can be reduced by narrowing it down to a smaller number of attributes. Another measure is to make a decision through a non-compensatory form of lexicographic or conjunctive (satisfactory) method while considering multiple attributes rather than an additive method. The information load might be reduced by avoiding a complex combination of information across multiple attributes to make decisions in a simple manner.

Unlike the conventional multi-attribute utility theory, this chapter did not assume the possibility of tradeoffs among the attributes in advance when normatively considering multi-attribute decision-making. As suggested by Arrow's general possibility theorem (1951), however, we found that rational decision-making would be impossible without the assumption of attribute tradeoffs unless the decision-making is based on one-dimensional standards. Conversely, both rational and appropriate standards of decision-making could be satisfied only allowing attribute tradeoffs. For the types of tradeoffs that should be performed, experts of multi-attribute utility theory are proposing superior techniques. Attribute tradeoffs are extremely important, at least prescriptively, despite the unknown extent of justification allowed from a normative perspective.

The author, however, is uncertain as to whether it is really appropriate to perform tradeoffs so easily for any type of decision-making problem. For example, the problem should be solved easily if a decision is made by trading off human life for economic efficiency to determine a price per person. However, making a decision based on such a scheme itself is unethical and might not necessarily result in a "good decision." Considering that alternatives across multiple attributes is important to prevent simple, one-dimensional decision-making.

In the field of normative decision theory related to what constitutes good decision-making, the conventional normative decision theory often considered "goodness" in view of the rationality of form. This chapter has exemplified from the formal perspective that multi-attribute decision-making satisfies such rationality criteria as transitivity and connectivity and that conditions deemed appropriate in multi-attribute decision-making contradict when multi-attribute decision-making is viewed from the above formal perspective. By interpreting Arrow's general possibility theorem (1951), one finds that rational decision-making is possible only when it is based on one-dimensional standards. Additionally, in view of behavioral decision theory, which is the descriptive decision theory of the actual decision-making process, decision-making based on one-dimensional standards occurs easily; pluralistic decision-making tends to be avoided attributable to the nature of people's attention (Takemura 2011a, b) Yet for decision-making for which the result is important, making the decision based only on one value is extremely dangerous considering that people hold multidimensional values. To produce a "good decision," it is important to regard the decision comprehensively by focusing on a plurality of values. From a prescriptive or normative perspective, it is important to consider values pluralistically and attempt to combine or trade off different values. Applying such a perspective might result in the abandonment of consistent, rational decision-making based on utility maximization. The pluralistic perspective, nonetheless, is considered necessary for good decision-making. Use of formal rationality in any manner would not help solve a problem of this type. Moreover, the actual social research or descriptive surveys of actual conditions such as psychological experiments alone would not help find the answer. In decision theory, or even in health psychology or social psychology, extremely normative questions such as "what is a good way of living a life?" or "what is a good society?" could not be avoided once the path to seek good decision-making is chosen. Psychology is a descriptive study and idealistic argument is strictly prohibited in psychology education. This might reflect tragic experiences in human history caused by idealistic stances that are not based on facts. Although such trends offer some positive effects, the purpose of behavioral decision research would be unclear if it only aimed at description. Normative examination is important also for descriptive research. Descriptive research is necessary for normative research. Similarly, prescriptions for actual decision-making cannot be developed solely from normative examination. As hinted by Aristotle, whether using a normative approach, descriptive approach, or prescriptive approach, decision research should ultimately pursue the answer to the question of what a good decision is. Aiming only at description, however, would obscure the goal of decision research. Decision research, therefore, must begin with consideration of what a good decision is.

References

Aristotle (1971). *Nikomakosu Rinrigaku* [Trans. by S. Takada (Ed.), *Nichomachean ethics*]. Tokyo: Iwanami Shoten.

Arrow, K. J. (1951). *Social choice and individual values*. New York: Wiley.

Berlin, I. (1969). *Four essays on liberty*. London: Oxford University Press.

Berlin, I. (1990). *The crooked timber of humanity: Chapters in the history of ideas is the fifth in a series of essay collections by Isaiah Berlin, compiled and edited by Henry Hardy*. London: John Murray.

Brandstätter, E., Giggerenzer, G., & Hertwig, R. (2006). The priority heuristic: Making choices without trade-offs. *Psychological Review, 113*, 409–432.

Carmone, F. J., Green, P. E., & Jain, A. K. (1978). Robustness of conjoint analysis: Some Monte Carlo results. *Journal of Marketing Research, 15*, 300–303.

Cattin, P., & Wittink, D. R. (1989). Commercial use of conjoint analysis: An update. *Journal of Marketing, 53*, 91–96.

Crowder, G. (1994). Pluralism & liberalism. *Political Studies, 42*, 293–305.

Crowder, G. (2002). *Liberalism and value pluralism*. London: Continuum.

Fujii, S., & Takemura, K. (2001a). Risuku taido to chui : Jokyo izonteki shoten moderu ni yoru furemingu koka no keiryo bunseki [Risk attitude and attention: A psychometric analysis of framing effect by contingent focus model]. *Kodo keiryogaku [The Japanese Journal of Behaviormetrics], 28*, 9–17.

Fujii, S., & Takemura, K. (2001b). Jokyo izonteki shoten moderu ni yoru furemingu koka no meta bunseki [Psychometric meta-analysis of framing effect by contingent focus model] Nihon kodo keiryo gakkai dai 29 kai taikai happyou ronbun syorokusyu [Paper Presented at the 29th International Meeting of the Psychometric Society (IMPS 2001)] (pp. 164–167).

Fujii, S., & Takemura, K. (2003). Attention, frames condition and decision making under risk: An empirical test of the contingent focus model using an eye gaze recorder. *Paper presented at the Society for Judgment and Decision Making*, Vancouver, Canada.

Fujii, S., Takemura, K., & Kikkawa, T. (2002). Kimekata to goi keisei: Shakaiteki jiremma ni okeru rikoteki doki no yokusei ni mukete [Decision strategy and consensus formation: Towards inhibition of selfish motive in social dilemma]. *Dobokugakkai rombunshu [Proceedings of Japan Society of Civil Engineers], 709*, 13–26.

Gigerenzer, G., & Goldstein, D. (1996). Reasoning the fast and frugal way: Models of bounded rationality. *Psychological Review, 103*, 650–669.

Holyoak, K. J., & Thagard, P. (1995). *Mental leaps: Analogy in creative thought*. Cambridge: MIT.

Ideno, T., Okubo, S., Tamari, Y., Abe, S., & Takemura, K. (2012). Ishi kettei katei ni kansuru shitsumon-shi shakudo no kaihatsu [Development of questionnaire of decision making process]. *The 16th Experimental Social Science Conference*, Tokyo, Japan

Karmakar, U. S. (1978). Subjective weighted utility: A descriptive extension of the expected utility model. *Organizational Behavior and Human Performance, 21*, 61–72.

Kojima, S. (1959). *Shohisha shinri no kenkyu*. Nihon Seisansei Honbu [*Consumer Psychology*]. Japan Productivity Center.

Kojima, S. (1994). Psychological approach to consumer buying decisions: Analysis of the psychological purse and psychology of price. *Japanese Psychological Research, 36*, 10–19.

Krantz, D. H., Luce, R. D., Suppes, P., & Tversky, A. (1971). *Foundations of measurement volume 1: Additive and polynomial representations*. New York: Academic.

Louviere, J. J. (1988). *Analyzing decision making: Metric conjoint analysis*. Newbury: Sage.

Luce, R. D., & Tukey, J. W. (1964). Simultaneous conjoint measurement: A new type of fundamental measurement. *Journal of Mathematical Psychology, 1*, 1–27.

Montgomery, H. (1983). Decision rules and the search for a dominance structure: Towards a process model of decision-making. In P. C. Humphreys, O. Svenson, & A. Vari (Eds.), *Analyzing and aiding decision processes* (pp. 343–369). Amsterdam: North-Holland.

Montgomery, H. (1993). The search for a dominance structure in decision-making: Examining the evidence. In G. A. Klein, J. Orasanu, R. Calderwood, & C. E. Zsambok (Eds.), *Decision-making in action: Models and methods* (pp. 182–187). Norwood: Ablex.

Okubo, S., & Takemura, K. (2011). Shirizu shohisha kodo to maketeingu 2 gankyu undo sokutei to shohisha kodo [Consumer behavior and marketing (2) measurement of the eye movement and consumer behavior]. *Seni seihin shohi kagaku [Journal of the Japan Research Association for Textile End-uses], 52*(12), 744–750.

Prelec, D. (1998). The probability Weighting Function. *Econometrica*, *66*(3), 497–527.

Schwartz, B., Ward, A., Monterosso, J., Lyubomirsky, S., White, K., & Lehman, D. R. (2002). Maximizing versus satisficing: Happiness is a matter of choice. *Journal of Personality and Social Psychology, 83*, 1178–1197.

Selart, M. (1997). Aspects of compatibility and construction of preference. In R. Ranyard, W. R. Crozier, & O. Svenson (Eds.), *Decision making: Cognitive models and explanations* (pp. 58–71). London: Routledge.

Shepard, R. N., Romney, A. K., & Nerlove, S. (Eds.). (1972). *Multidimensional scaling* (Vol. I). New York: Seminar.

Takahashi, N., Takemura, k., Ideno, T., Okubo, S., & Tamari, Y. (2010). Aimai Jitai Ni Okeru Keishikisei Tsuikyu Keiko Ga Soshikinai Deno Ihan Ni Taisuru Ishiki To Shakai Handan Ni Ataeru Eikyo [Effect of tendency to seek formality in ambiguous situations on the awareness of violation in an organization and social judgment]. *Presentation at the 51st Conference of the Japanese Society of Social Psychology* (pp. 762–763) (In Japanese).

Takemura, K. (1994). The contingent focus model of the framing effect. *Japanese Psychological Review, 37*(3), 270–193.

Takemura, K. (1996). *Ishikettei no shinri – sono katei no tankyu, Fukumura Shuppan [Psychology of decision-making: Investigation of its process]*. Tokyo: Fukumura Publishing.

Takemura, K. (1998). Jokyo izonteki isikettei no teiseiteki moderu: Shinteki monosashi riron niyoru setsumei [Qualitative model of contingent decision-making: An explanation of using the mental ruler theory]. *Ninchi Kagaku (Cognitive Studies), 5*(4), 1–18.

Takemura, K. (2001). Contingent decision making in the social world. In C. M. Allwood & S. Selart (Eds.), *Decision-making: Social and creative dimensions* (pp. 153–173). Boston: Kluwer Academic.

Takemura, K. (2011a). Tazokusei ishikettei no shinri moderu to yoi ishikettei [Model of multi-attribute decision making and good decision]. *Opereshonzu risachi [Operations Research], 56*(10), 583–590 (in Japanese).

Takemura, K. (2011b). Shirizu shohisha kodo to maketeingu (1) shohisha no tazokusei ishikettei to sono bunseki [Consumer behavior and marketing (1) consumer decision-making by other attributes and its analysis]. *Seni seihin shohi kagaku [Journal of the Japan Research Association for Textile End-uses], 52*(11), 670–677.

Takemura, K., & Fujii, S. (2014). Ishikettei no shoho-Jokyo izon syoten moderu no tenkai, Asakura Shoten [Prescription of decision making: Development of contingent focus model]. Tokyo: Asakura Publishing.

Takemura, K., Wakayama, D., & Horiuchi, K. (2004). Kokoku juyo no suri shinri moderu to deta kaiseki ho no kaihatsu: shohisha no handan to ishikettei no shinri jikken to chosa kenkyu wo tsujite [Development of mathematical model and data analysis method of consumer advertising response: Through psychological experiment and survey research]. *Kokoku Kagaku [Journal of Advertising Science (Japan Academy of Advertising)], 45*, 153–172 (in Japanese).

Tversky, A., & Fox, C. R. (1995). Weighting risk and uncertainty. *Psychological Review, 102*, 269–283.

Tversky, A., & Kahneman, D. (1981). The framing decisions and the psychology of choice. *Science, 211*, 453–458.

Tversky, A., & Kahneman, D. (1992). Advances in the prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5*, 297–323.

Tversky, A., & Wakker, P. (1995). Risk attitudes and decision weight. *Econometrica, 63*, 1255–1280.

Wu, G., & Gonzalez, R. (1996). Curvature of the probability weighting function. *Management Science, 42*, 1676–1690.

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Subject Index

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